3.20. A Markov source with output symbols $\{A, B, C\}$, is characterised by the graph in Figure 2.

(a) What is the stationary distribution for the source?

(b) Determine the entropy of the source, H_∞.

(c) Consider a memory-less source with the same probability distribution as the stationary distribution calculated in (a). What is the entropy for the memory-less source?

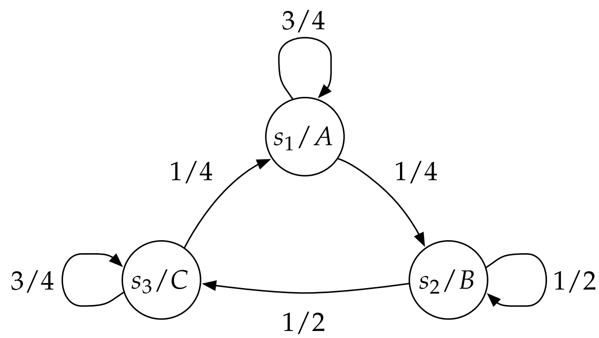

Figure 2: A Markov graph for the source in Problem **??**

Solution:

(a) The transition matrix is

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 0 & 3/4 \end{pmatrix}$$

The stationary distribution is found from

$$\boldsymbol{\mu} P = \boldsymbol{\mu}$$

$$\Rightarrow \begin{cases} -\frac{1}{4}\mu_1 & & +\frac{1}{4}\mu_3 = 0 \\ \frac{1}{4}\mu_1 & -\frac{1}{2}\mu_2 & = 0 \\ & \frac{1}{2}\mu_2 & -\frac{1}{4}\mu_3 = 0 \end{cases}$$

Together with $\sum_i \mu_i = 1$ we get $\mu_1 = \frac{2}{5}$, $\mu_2 = \frac{1}{5}$, $\mu_3 = \frac{2}{5}$.

(b) The entropy rate is

$$H_\infty(U) = \sum_i \mu_i H(S_i) = \frac{2}{5}h\Big(\frac{1}{4}\Big) + \frac{1}{5}h\Big(\frac{1}{2}\Big) + \frac{2}{5}h\Big(\frac{1}{4}\Big)$$
$$= \frac{4}{5}\Big(2 - \frac{3}{4}\log 3\Big) + \frac{1}{5} = \frac{1}{9} - \frac{3}{4}\log 3 \approx 0.8490$$

(c) $H\Big(\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\Big) = -\frac{2}{5}\log\frac{2}{5} - \frac{1}{5}\log\frac{1}{5} - \frac{2}{5}\log\frac{2}{5} = \log 5 - \frac{4}{5} \approx 1.5219$
That is, we gain in uncertainty if we take into consideration the memory of the source.