Information Theory

Lecture 8
Capacity of DMC

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Discrete memoryless channel (DMC)

Definition

A discrete channel is a system $(\mathcal{X}, P(y|x), \mathcal{Y})$, with

- input alphabet $\mathcal{X}$ of size N
- output alphabet $\mathcal{Y}$ of size M
- transition probability distribution $P(y|x)$

The channel is memoryless if the probability distribution is independent of previous input symbols.

Channel capacity

Definition

The information channel capacity of a DMC is

$$C = \max_{p(x)} I(X;Y)$$

where the maximum is taken over all input distributions.

Theorem (Channel Coding Theorem)

A code rate is achievable if and only if

$$R < C = \max_{p(x)} I(X;Y)$$

DMC

Definition

A discrete memoryless channel is uniformly dispersive if the distribution for the outgoing branches has the same set of probabilities.

Theorem

If a channel is uniformly dispersive, then

$$H(Y|X) = H(\boldsymbol{r})$$

where $\boldsymbol{r}$ *is the set of probabilities for outgoing branches. The capacity of the channel can be written*

$$C = \max_{p(x)} H(Y) - H(\boldsymbol{r})$$

DMC

Definition (Symmetric channels)

A discrete memoryless channel is symmetric if

- for all X, the branches leaving the code symbol have the same set of probabilities, $p_1, p_2, \dots, p_M$.
- for all Y, the branches entering the received symbol have the same set of probabilities, $p_1, p_2, \dots, p_N$.

Seen from the transition matrix $[p(y|x)]$ all rows are permutations of each other and all columns are permutations of each other.

DMC

Definition (Weakly symmetric channels)

A discrete memoryless channel is weakly symmetric if

- for all X, the branches leaving the code symbol have the same set of probabilities, $p_1, p_2, \ldots, p_M$.
- for all Y, the branches entering the received symbol have the same sum, $\sum_{i=1}^{N} p_i$.

Seen from the transition matrix $[p(y|x)]$ all rows are permutations of each other and all columns have the same sum $\sum_x p(y|x)$.

DMC

Theorem

If a channel is symmetric or weakly symmetric, the channel capacity is

$$C = \log M - H(\boldsymbol{r})$$

where $\boldsymbol{r}$ *is the set of probabilities labeling brances leaving a code symbol* X*, or, viewed in a transition matrix, one row of the transition matrix.*