# Hand in problem 3 in Information Theory (EITN45)

VT 2, 2017

## Problem 3

In the course material error correcting encoding and decoding is described through a Hamming code. There are many other codes defined over the years, and the one treated here is called Reed-Muller code. For a positive integer $m$ and a non-negative integer $r$, the generator matrix for the Reed-Muller code $\mathrm{RM}(r,m)$ is defined as

$$G(r,m) = \begin{pmatrix} G(r,m-1) & G(r,m-1) \\ 0 & G(r-1,m-1) \end{pmatrix}$$

To start the induction define $G(0,m)$ as a row vector of $2^m$ ones, i.e. the generator matrix for a length $2^m$ repetition code, and $G(m,m)$ the unit matrix of size $2^m$. In this problem the $\mathrm{RM}(1,3)$ code will be treated.

(a) Show that the above induction formula gives the generator matrix

$$G(1,3) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(b) List all codewords and determine $d_{\text{min}}$. How many errors can be corrected?

(c) Verify that $H = G$, i.e. that $GG^T = 0$. That means the generator matrix is its own parity check matrix, and such code is said to be self dual.[1]

(d) Write the syndrome table for all single errors that can occur on the channel.

(e) Choose a codeword and introduce one error in it. Decode using the syndrome table.

(f) Is it possible by using the syndrome table above to simultaneously

- ▶ correct all single errors
- ▶ detect all double errors

That is, do all double errors give syndromes different from syndromes for single errors or no errors?

[1]This is a special case for this RM code. In general the parity check matrix for a $\mathrm{RM}(r,m)$ code is given by the generator matrix $G(m-r-1,m)$.

## Hand in details

You should hand in your solution to the problem. The line of reasoning should be clear from the solution. If you use computer to solve parts of the problem you should also hand in the code for these scripts as appendix for the solutions.

The solutions should be handed in individually to Stefan or Umar, or in the course mailbox at the third floor in the northern staircase from the main entrance in the E-building. Do not forget to write your name and STIL or student ID.

## Tips and tricks in MATLAB

For those using MATLAB it can be good to know:

- An integer `a` is converted to its binary form by `dec2bin(a,n)` where `n` is the number of bits. Then the binary vector is a string. To convert back to a number you can use `dec2bin(a,n)-'0'`.
- Then the Hamming weight of the binary representation of 45 can be derived using `sum(dec2bin(45,8)-'0')`.
- To get the value of a numeric form binary vector you can use `bin2dec(num2str([0 1 0 0 1 1]))`.