LUND UNIVERSITY
Electrical and Information Technology

Home Exam in Information Theory, EITN45

May 26 – June 3, 2014

Name: ______________________________

Id Number: ______________________________

Programme: ____________

Nbr of sheets: ____________

Mark with a cross the problems you solved.				
1	2	3	4	5

Signature: ______________________________

Control protocol						
1	2	3	4	5	$\sum$	Grade

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- Problems shall be solved and handed in individually.
- Write your name on each paper.
- Start a new solution on a new sheet of paper. Use only one side of the paper.
- Solutions should clearly show the line of reasoning.
- Program code, e.g. MATLAB, python or C/C++, used for derivations should be handed in as a complement to the solution. However, the solutions should be fully understandable without the code. Normal command line calculations, e.g. entropy, is not required to hand in.

Good luck!

Problem 1

Consider the joint distribution for the random variables X and Y given below.

	$p(x,y)$	Y = 0	1	2	3
	0	1/4	0	0	0
	1	1/10	3/10	0	0
X	2	0	1/20	1/10	0
	3	0	0	1/20	1/10
	4	0	0	1/20	0

(a) Determine $H(X)$, $H(Y)$, $H(X|Y)$, $H(Y|X)$ and $H(X,Y)$.

(b) Assume the distribution is given for the transmitted and received variables, X and Y, over the channel in Figure 1.1. Then this corresponds to a given distribution over X. Show that this is not a capacity generating distribution.

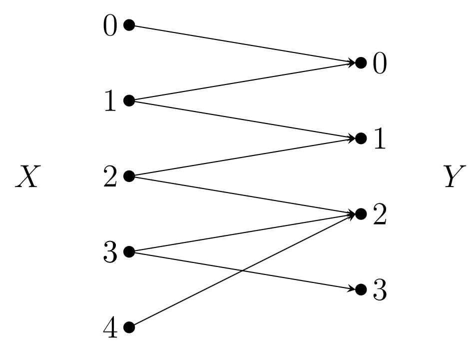

Figure 1.1: A channel with five inputs and four outputs

All derivations should be clearly stated. You are allowed to use the function Entropy.m and MutualInformation.m from Hand in 1 without specifying the code.

(5+5=10p)

Problem 2

Use LZ77 with $S = 31$ and $B = 15$ to compress the text

> There are two minutes difference from four to two to two to two,
> and from two to two to two, too

Initialise the search buffer with the first S letters of the text. The result should be given as codewords in the form (j, l, c), i.e. not in binary form. You should also find the compression ratio[1] , $R = L_u/L_c$, when assuming that the characters are encoded with 8 bits each.

(10p)

[1] Right now there is some confusion in the book about this, but it should be uncompressed length divided by the compressed length.

Problem 3

Consider six parallell Gaussian channels with the noise levels

$$N = (8, 12, 14, 10, 16, 6)$$

The total allowed power usage in the transmitted signal is $P = 19$.

(a) What is the capacity of the combined channel?

(b) If you must divide the power equally over the six channels, what is the capacity?

(c) If you decide to use only one of the channels, what is the maximum capacity?

(5+3+2=10p)

Problem 4

Consider the channel in Figure 4.1. Both the input and the noise are binary random variables, $X \in \{0, 1\}$ and $Z \in \{0, 1\}$. The distribution of the multiplied noise is $P(Z = 0) = \alpha$. The output from the channel is the multiplication of the input and the noise, $Y = X \cdot Z$. Derive an expression for the capacity of the channel. Make a sketch plot of the capacity as a function of the error probability α.

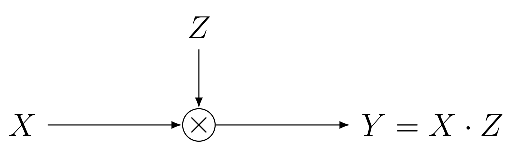

Figure 4.1: A discrete channel where the input and the noise is multiplied.

(10p)

Problem 5

A man climbs an infinitely long ladder. At each time instant he tosses a coin. If he gets Head he takes a step up on the ladder but if he gets Tail he drops down to the ground (step 0). The coin is counterfeit with $P(\text{Head}) = p$ and $P(\text{Tail}) = 1 - p$. The sequence of where on the ladder the man stands forms a Markov chain.

(a) Construct the state transition matrix for the process and draw the state transition graph.

(b) What is the entropy rate of the process?

(c) After the man has taken many steps according to the process you call him and ask if he is on the ground. What is the uncertainty about his answer? If he answers that he is not on the ground, what is the uncertainty of which step he is on? (You can trust that he is telling the truth.)

(2+3+5=10p)