Matched filtering

Transmitted complex baseband signal equals

$$y^c(t) = \sum_k a_k p(t - kT_s)$$

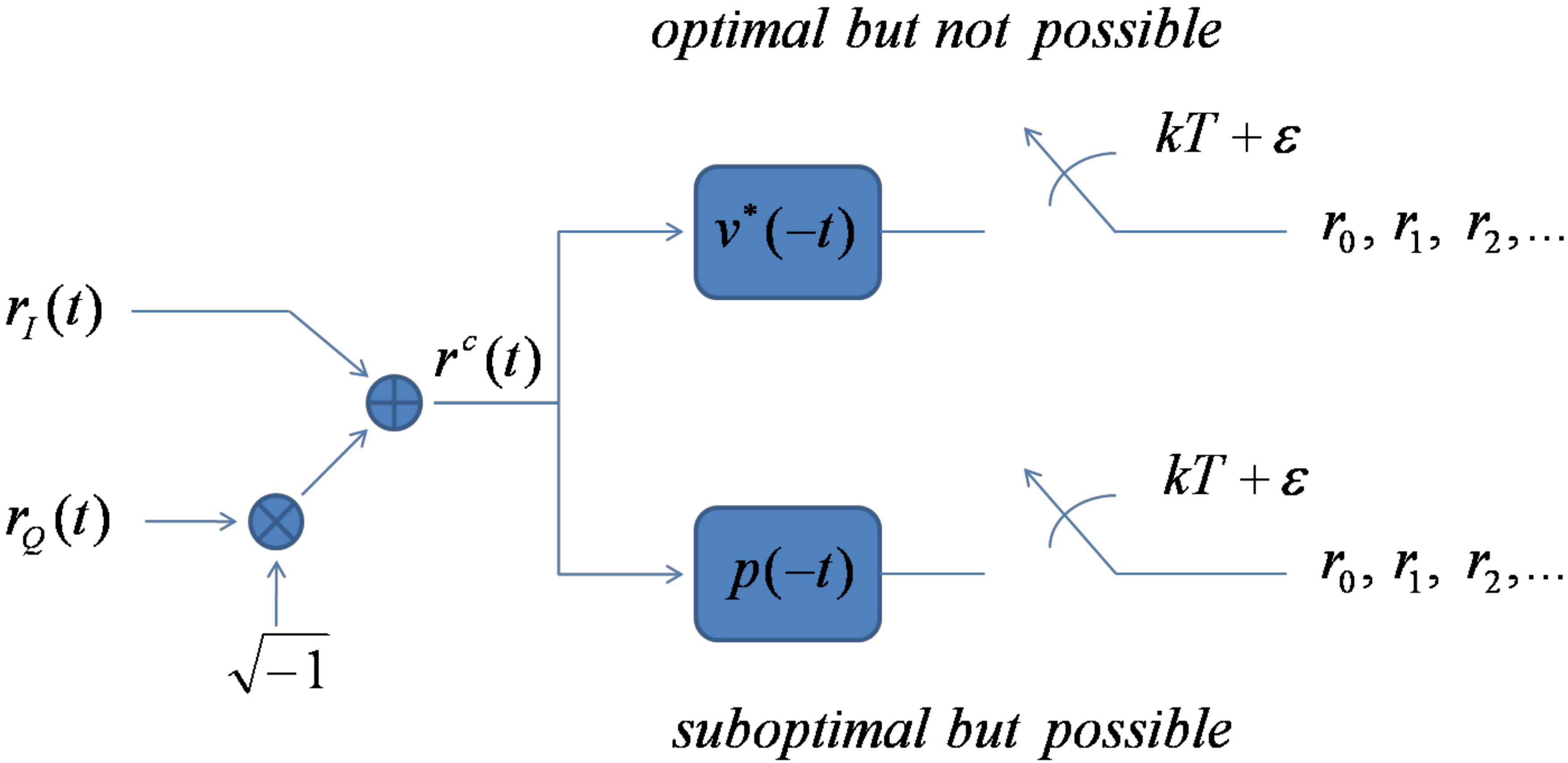
Where {a} are complex data symbols and p(t) any pulse shape.

Received complex baseband signal equals

$$r^c(t) = \sum_k a_k v(t - kT_s) + n^c(t) \text{ where } v(t) = p(t) \star h^c(t)$$

What is the optimal receiver processing?
We know that it is ”matched filtering”, but to what whould we match the filter?

Matched filtering



New problems emerge…..we don't know where to sample, this we model through the offset ε

ISI

Let $z(t)$ denote the matched filter. The signal part of each sample r_k equals

$$\begin{aligned}
r_k &= \left.\int_{-\infty}^{\infty} r^c(\tau) z(t-\tau)\mathrm{d}\tau\right|_{t=kT+\epsilon} \\
&= \left.\int_{-\infty}^{\infty} r^c(\tau) p(\tau - t)\mathrm{d}\tau\right|_{t=kT+\epsilon} \\
&= \int_{-\infty}^{\infty} \left(\sum_{\ell} a_\ell v(\tau - \ell T)\right) p(\tau - kT - \epsilon)\mathrm{d}\tau \\
&= \sum_{\ell} a_\ell \int_{-\infty}^{\infty} v(\tau - \ell T) p(\tau - kT - \epsilon)\mathrm{d}\tau \\
&= \sum_{\ell} a_\ell \int_{-\infty}^{\infty} v(\tau) p(\tau + (\ell - k)T - \epsilon)\mathrm{d}\tau \\
&= \sum_{\ell} a_\ell h_{k-\ell},
\end{aligned}$$

where

$$h_n \triangleq \int_{-\infty}^{\infty} v(\tau) p(\tau - nT - \epsilon)\mathrm{d}\tau.$$

ISI, Example 1

Example 1. Assume that $\epsilon = 0$, and that $h(t) = \delta(t)$, (i.e. no channel and no sampling mismatch). This implies that $h_I(t) = \delta(t)$ and $h_Q(t) = 0$ (can be realized from $\tilde{r}_I(t) = y_I(t)/2$ and $\tilde{r}_I Q = y_Q(t)/2$.) Hence, the complex channel equals,

$$h^c(t) = \delta(t)\cos(\phi) - \jmath\delta(t)\sin(\phi)$$

and $v(t)$ becomes

$$v(t) = p(t)\cos(\phi) - \jmath p(t)\sin(\phi).$$

Since $\epsilon = 0$ we get that $h_n = 0$ if $n \neq 0$. For $n = 0$ we get,

$$h_0 = \int p(t)^2 \mathrm{d}t \exp(-\jmath\phi) = E_p \exp(-\jmath\phi),$$

and we get that

$$r_k = E_p a_k \exp(-\jmath\phi) + \eta_k,$$

where η_k is a noise variable.

ISI, Example 2

Example 2. Assume that both ϕ and ϵ are non-zero, and that $h^c(t) \neq 0, 0 \leq t \leq T$.

It follows that $v(t)$ is of length $2T$ and may have a complicated shape that is not known to us.

How many non-zero ISI taps do we get?

Start by the case $\epsilon = 0$, then

$$\begin{aligned}
h_{-1} &= \int_0^{2T} v(t)p(t+T)\mathrm{d}t = 0 \\
h_0 &= \int_0^{2T} v(t)p(t)\mathrm{d}t \neq 0 \\
h_1 &= \int_0^{2T} v(t)p(t-T)\mathrm{d}t \neq 0 \\
h_2 &= \int_0^{2T} v(t)p(t-2T)\mathrm{d}t = 0.
\end{aligned}$$

Thus, for $\epsilon = 0$ there are two taps.

ISI, Example 2

Let us now move on to $\epsilon < 0$

$$\begin{aligned}
h_{-1} &= \int_0^{2T} v(t)p(t+T-\epsilon)\mathrm{d}t = 0 \\
h_0 &= \int_0^{2T} v(t)p(t-\epsilon)\mathrm{d}t \neq 0 \\
h_1 &= \int_0^{2T} v(t)p(t-T-\epsilon)\mathrm{d}t \neq 0 \\
h_2 &= \int_0^{2T} v(t)p(t-2T-\epsilon)\mathrm{d}t \neq 0 \\
h_3 &= \int_0^{2T} v(t)p(t-3T-\epsilon)\mathrm{d}t = 0.
\end{aligned}$$

Now take $\epsilon > 0$

$$\begin{aligned}
h_{-2} &= \int_0^{2T} v(t)p(t+2T-\epsilon)\mathrm{d}t = 0 \\
h_{-1} &= \int_0^{2T} v(t)p(t+T-\epsilon)\mathrm{d}t \neq 0 \\
h_0 &= \int_0^{2T} v(t)p(t-\epsilon)\mathrm{d}t \neq 0 \\
h_1 &= \int_0^{2T} v(t)p(t-T-\epsilon)\mathrm{d}t \neq 0 \\
h_2 &= \int_0^{2T} v(t)p(t-2T-\epsilon)\mathrm{d}t = 0.
\end{aligned}$$

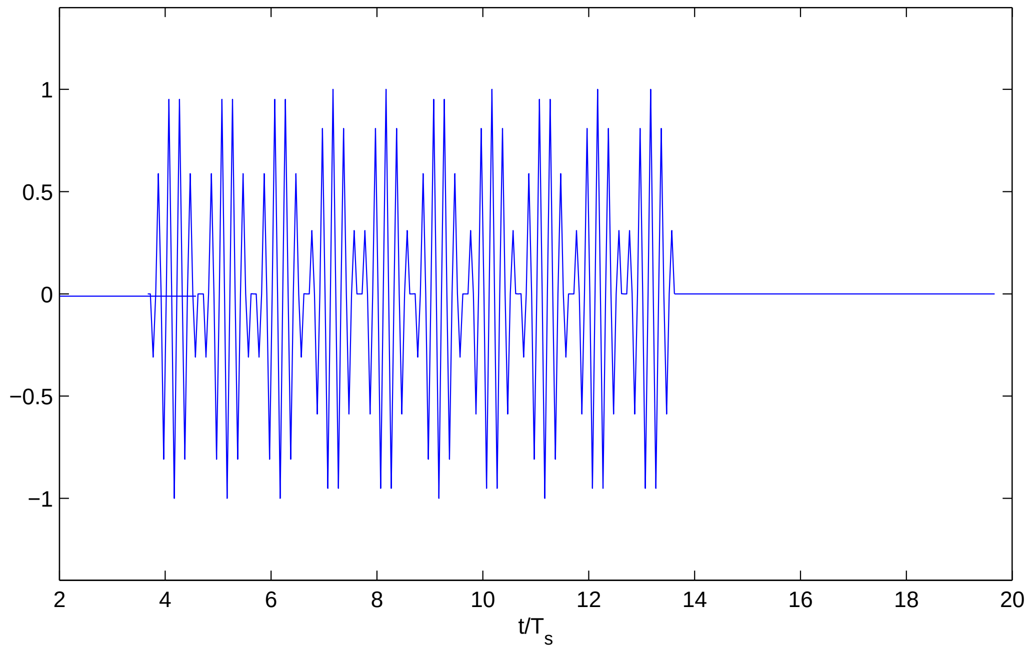
Thus, for $\epsilon \neq 0$ there are three taps.

Synchronization

Major problem: We dont know where the signal starts!

Synchronization

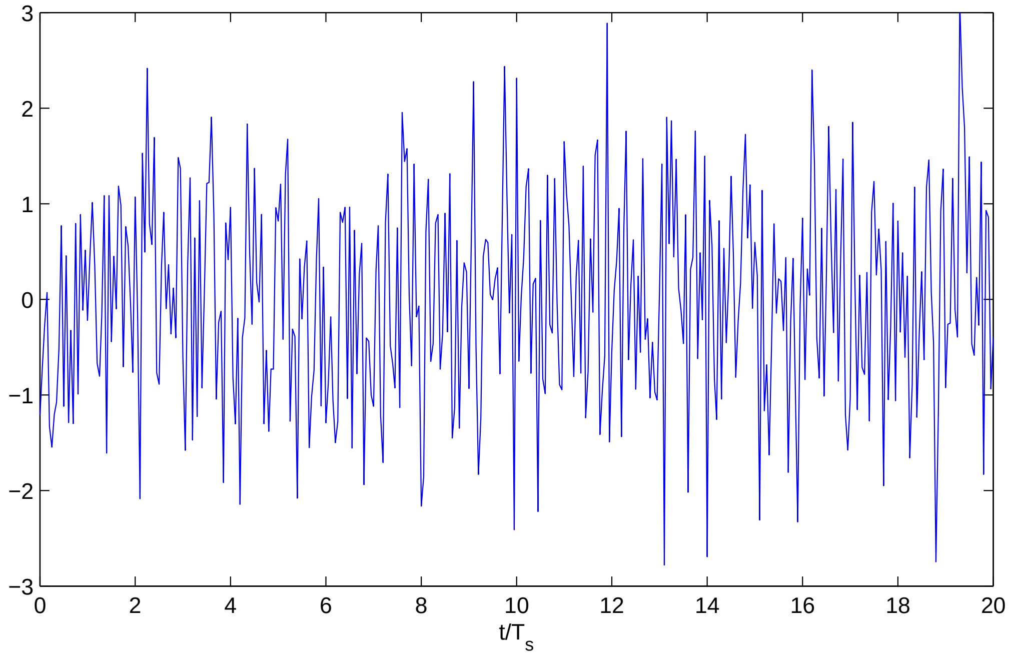
Major problem: We dont know where the signal starts!



If there is no noise, its easy....

Synchronization

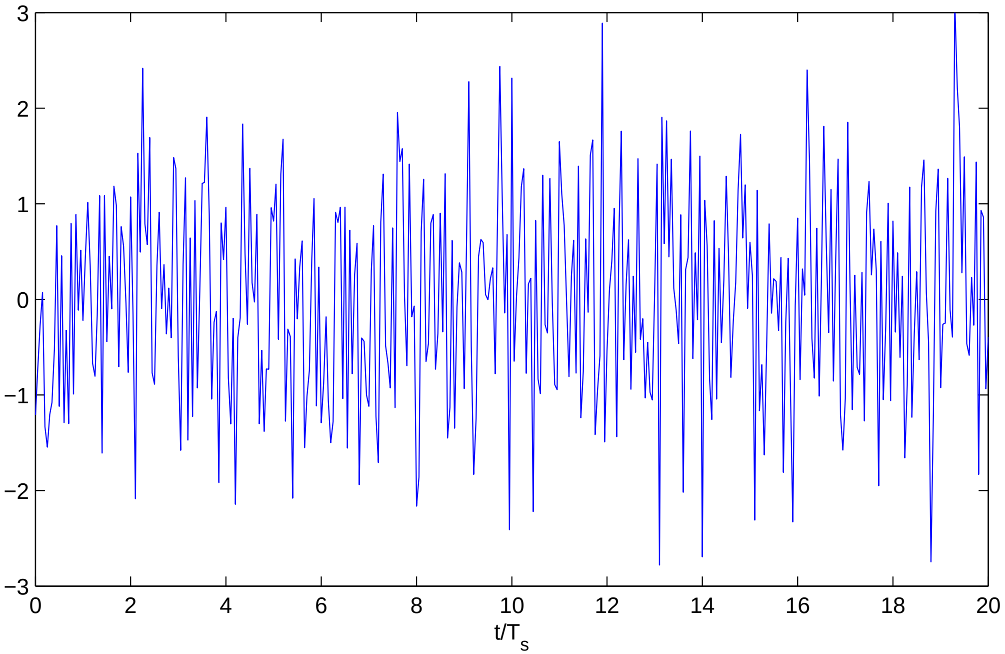
Major problem: We dont know where the signal starts!



How about this one?

Synchronization

Major problem: We dont know where the signal starts!



How about this one?
The signal starts at $t/T_s = 6.5$ and ends at $t/T_s = 16.5$. $E_b/N_0 = 10$ dB.

Synchronization

- In general, syncronization is a difficult subject
- Requires solid background in statistical signal processing and control theory
- Phase-locked-loops (PLL) are essential
- We will only make use of simple techniques....

Synchronization

Major problem: We dont know where the signal starts!

Not knowing ϕ will lead to a leakage between the in-phase and the quadrature components, but this can be neglected.

Not knowing ϵ is more severe and will be analyzed next.

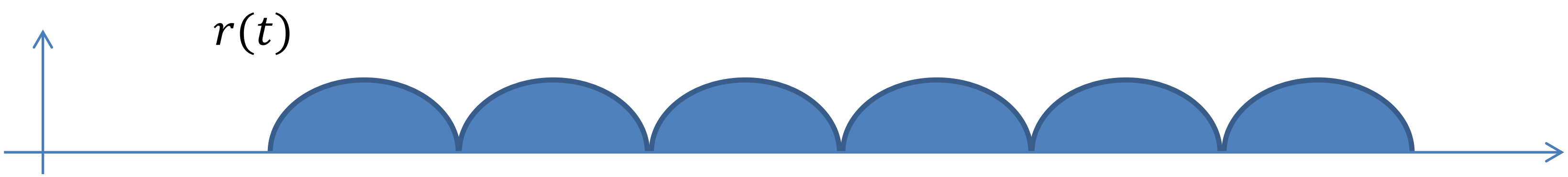
Synchronization

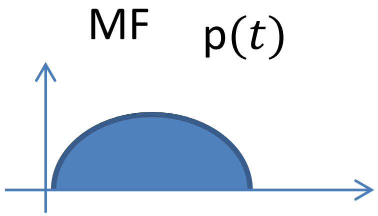
Task 2: Only mild channel is present, simple synchronization is enough. Will be analyzed as if the channel was not present at all

Task 3: More severe channel is present, and more advanced synchronization is needed. Will be treated after easter since there is a clear relation to OFDM

Synchronization in PWC 1

In-phase signal at the input to the MF

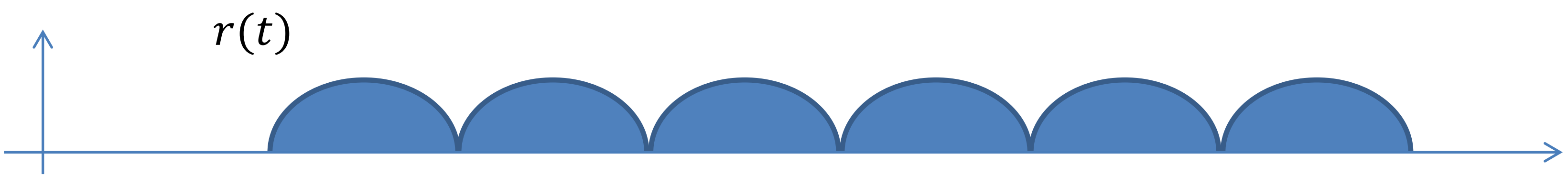


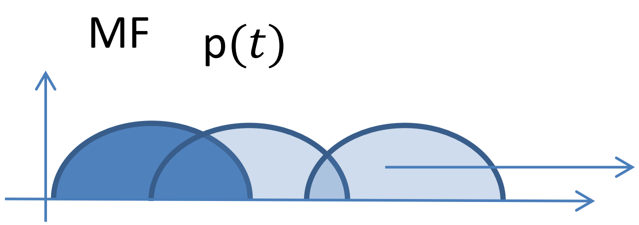


Operation of the MF: $\int p(\tau) r(t-\tau) d\tau$

Synchronization in PWC 1

In-phase signal at the input to the MF





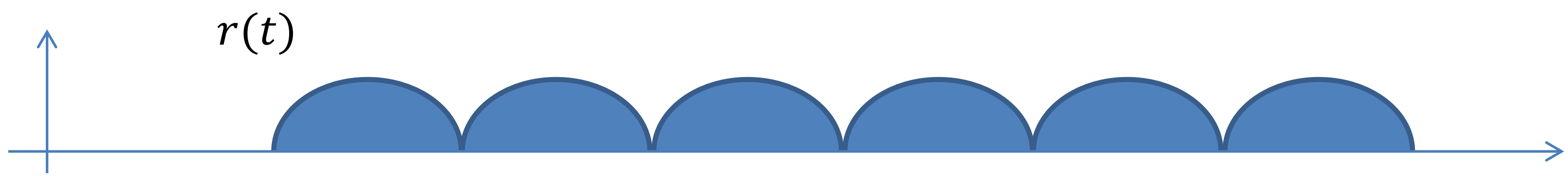
Operation of the MF: $\int p(\tau) r(t-\tau) d\tau$

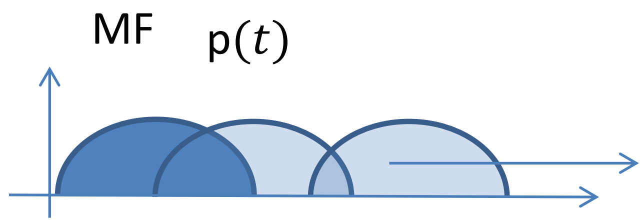
Graphical interpretation:

1. Let p(t) slide along the x-axis
2. At each position, multiply p(t) and r(t)
3. Integrate the product

Synchronization in PWC 1

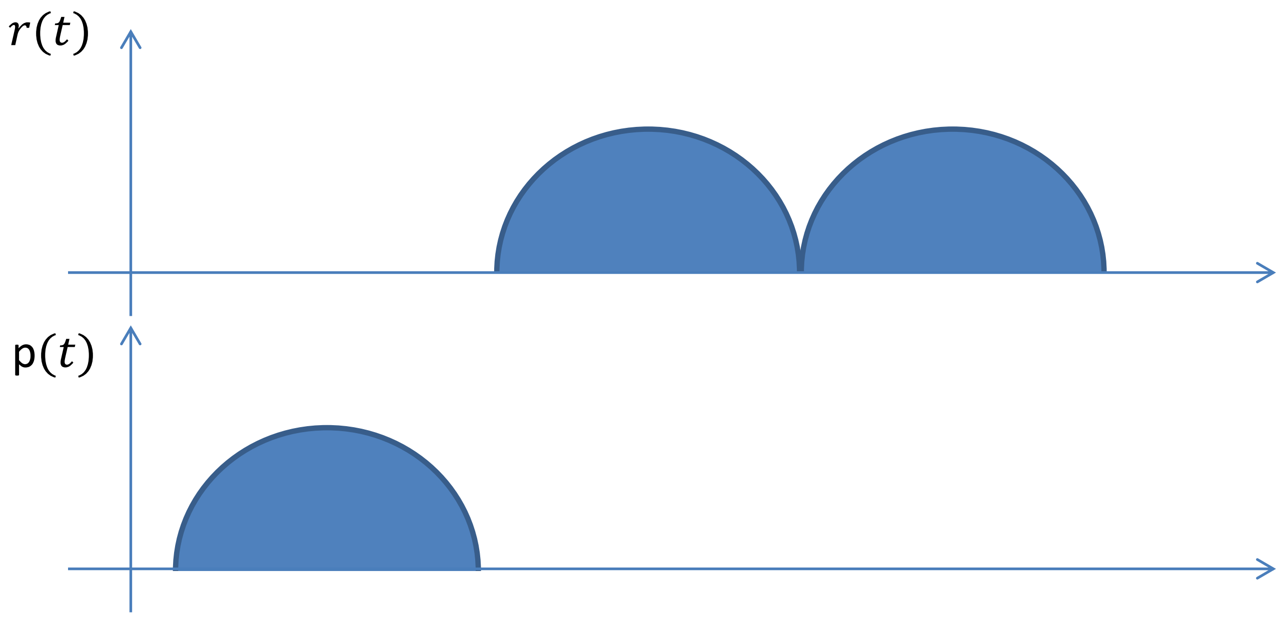
In-phase signal at the input to the MF





Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int \ (f(x))^2 dx}\sqrt{\int \ (g(x))^2 dx}$

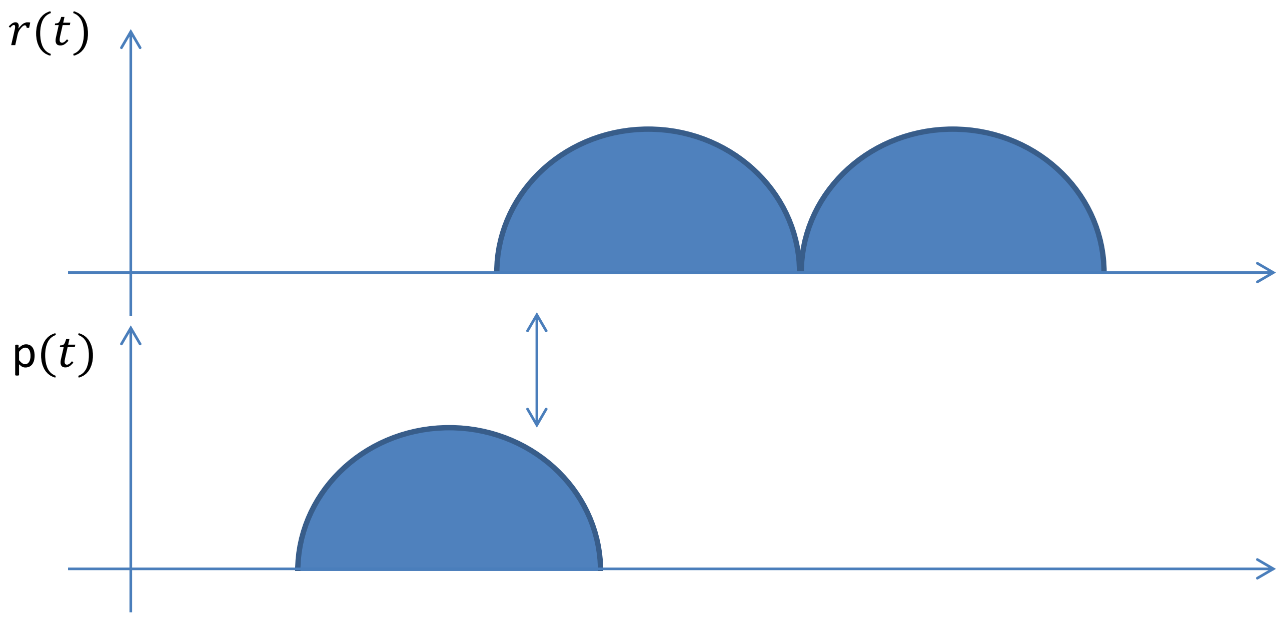
Synchronization in PWC 1



Only noise here

Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int (f(x))^2 dx}\sqrt{\int (g(x))^2 dx}$

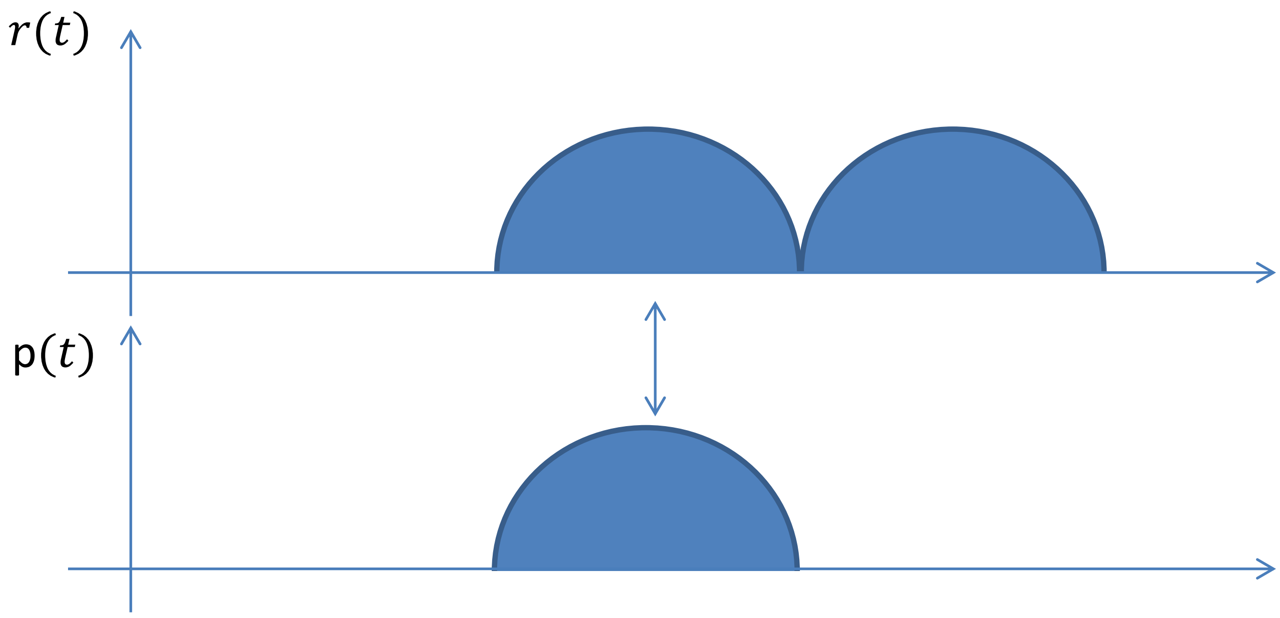
Synchronization in PWC 1



Some overlap here

Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int (f(x))^2 dx}\sqrt{\int (g(x))^2 dx}$

Synchronization in PWC 1



Fully overlaped. Max value due to C-S-inequality

Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int\ (f(x))^2dx}\sqrt{\int\ (g(x))^2dx}$

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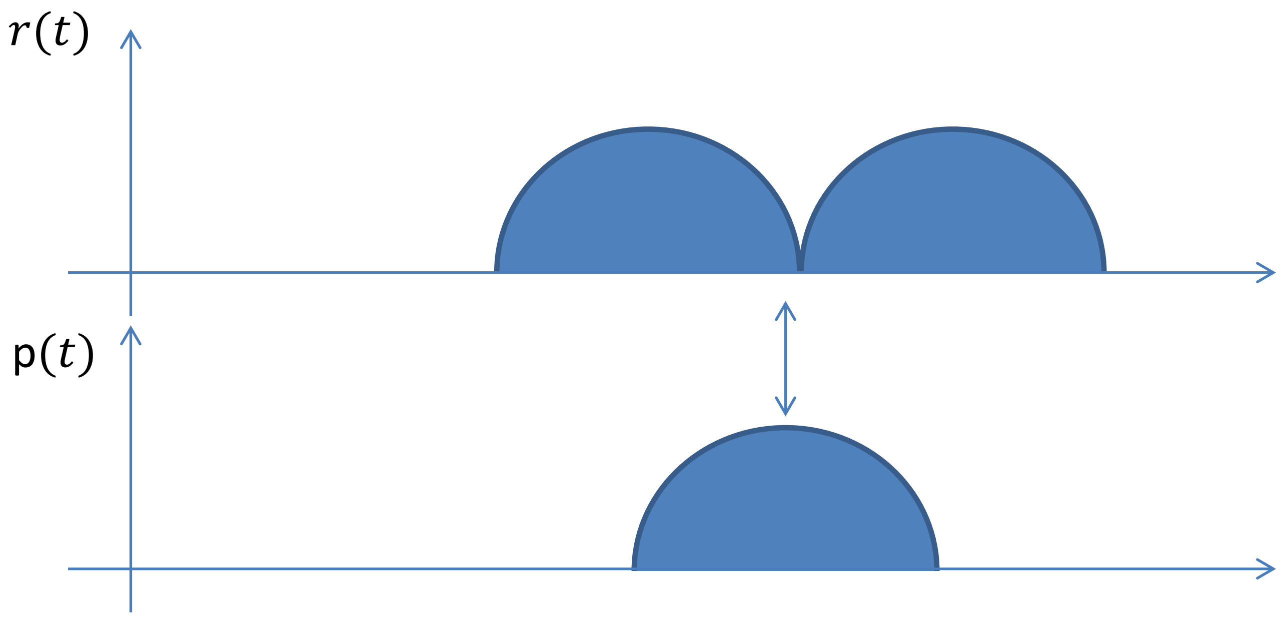
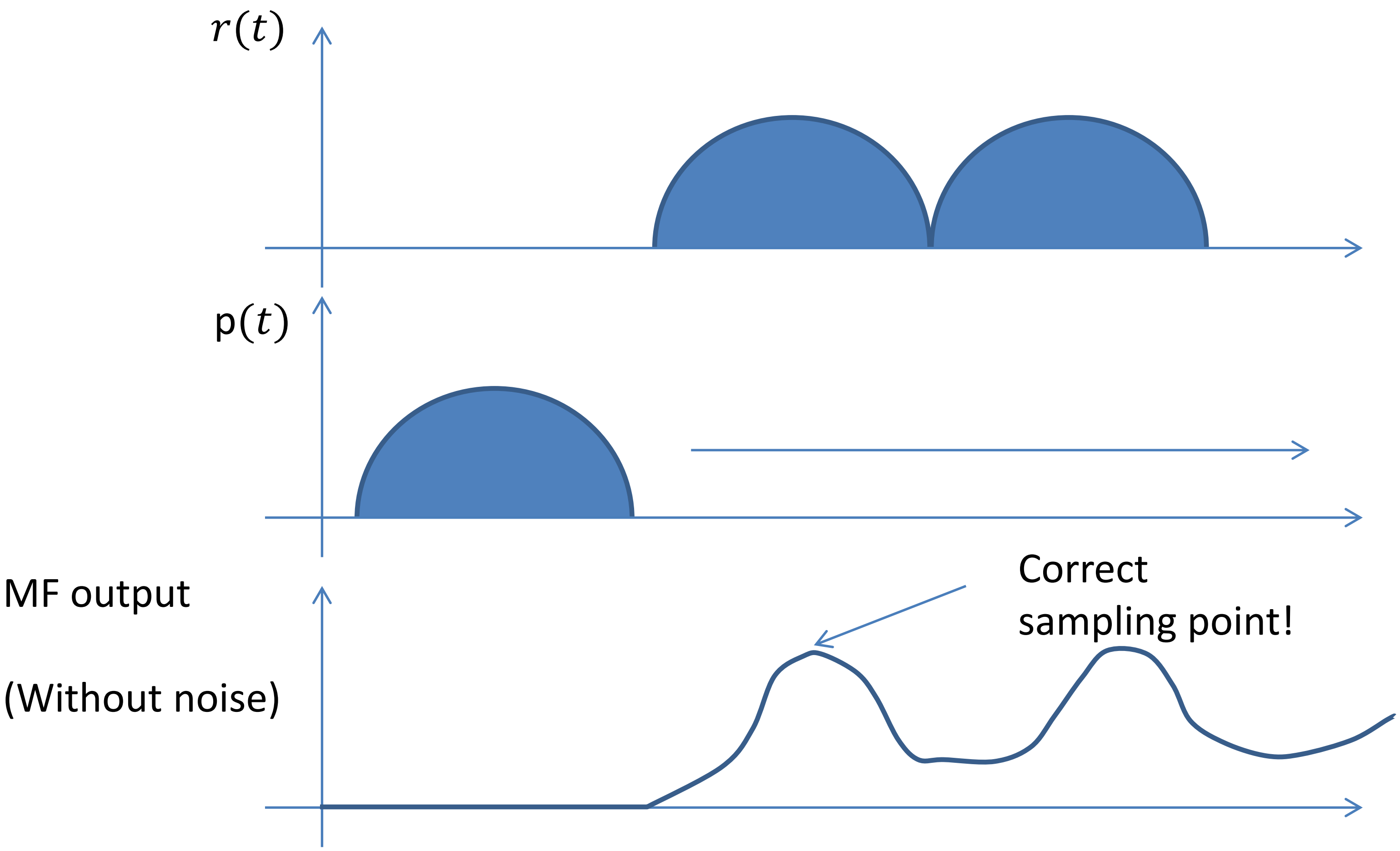
Synchronization in PWC 1



Less overlap again

Recall Cauchy-Schwarz: $|\int f(x)g(x)dx| \leq \sqrt{\int (f(x))^2 dx}\sqrt{\int (g(x))^2 dx}$

Synchronization in PWC 1



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Synchronization

The phase mismatch and non-optimal sampling instance yields a channel model

$$r[k] = \alpha e^{i\phi} a[k] + n[k]$$

Phase mismatch gives ϕ and sampling mismatch gives α

ϕ can be estimated from a pilot symbol p. Let

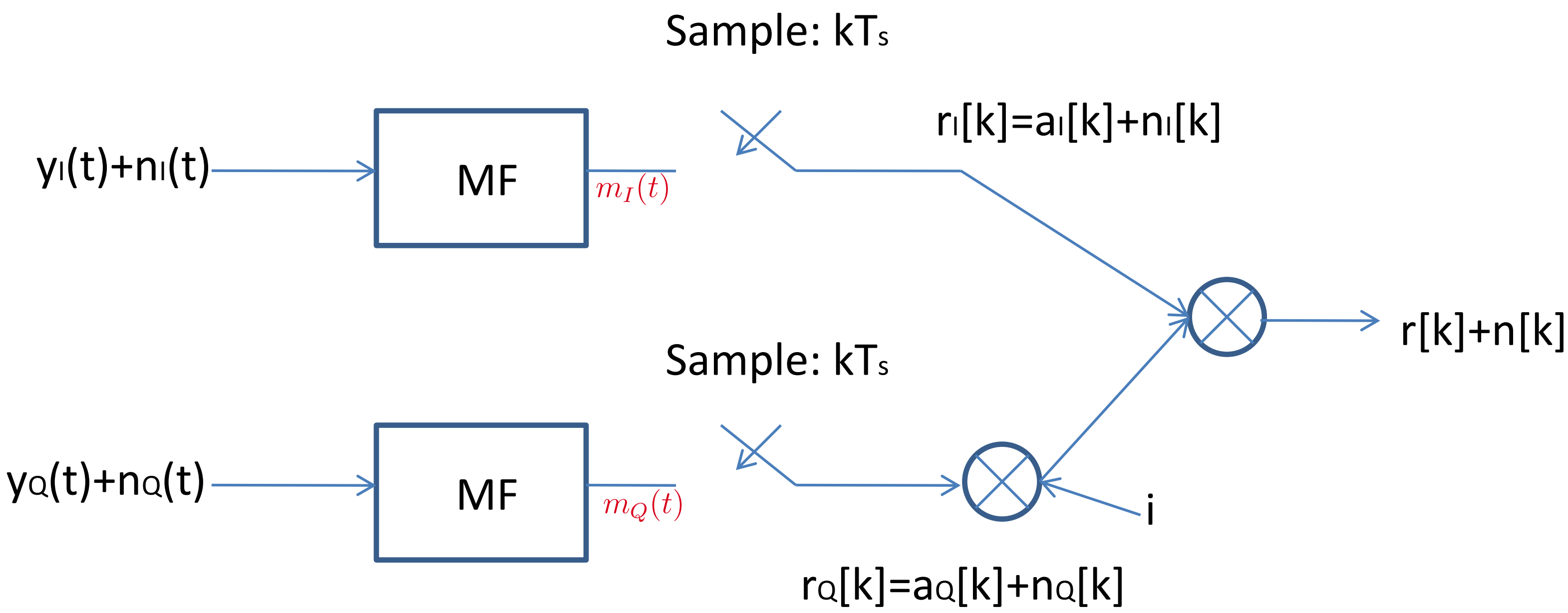
$$a[1] = p = 1 + i = \sqrt{2}e^{i\pi/4}$$

then

$$\hat{\phi} = \text{angle}\{r[1]\} - \frac{\pi}{4}$$

Synchronization

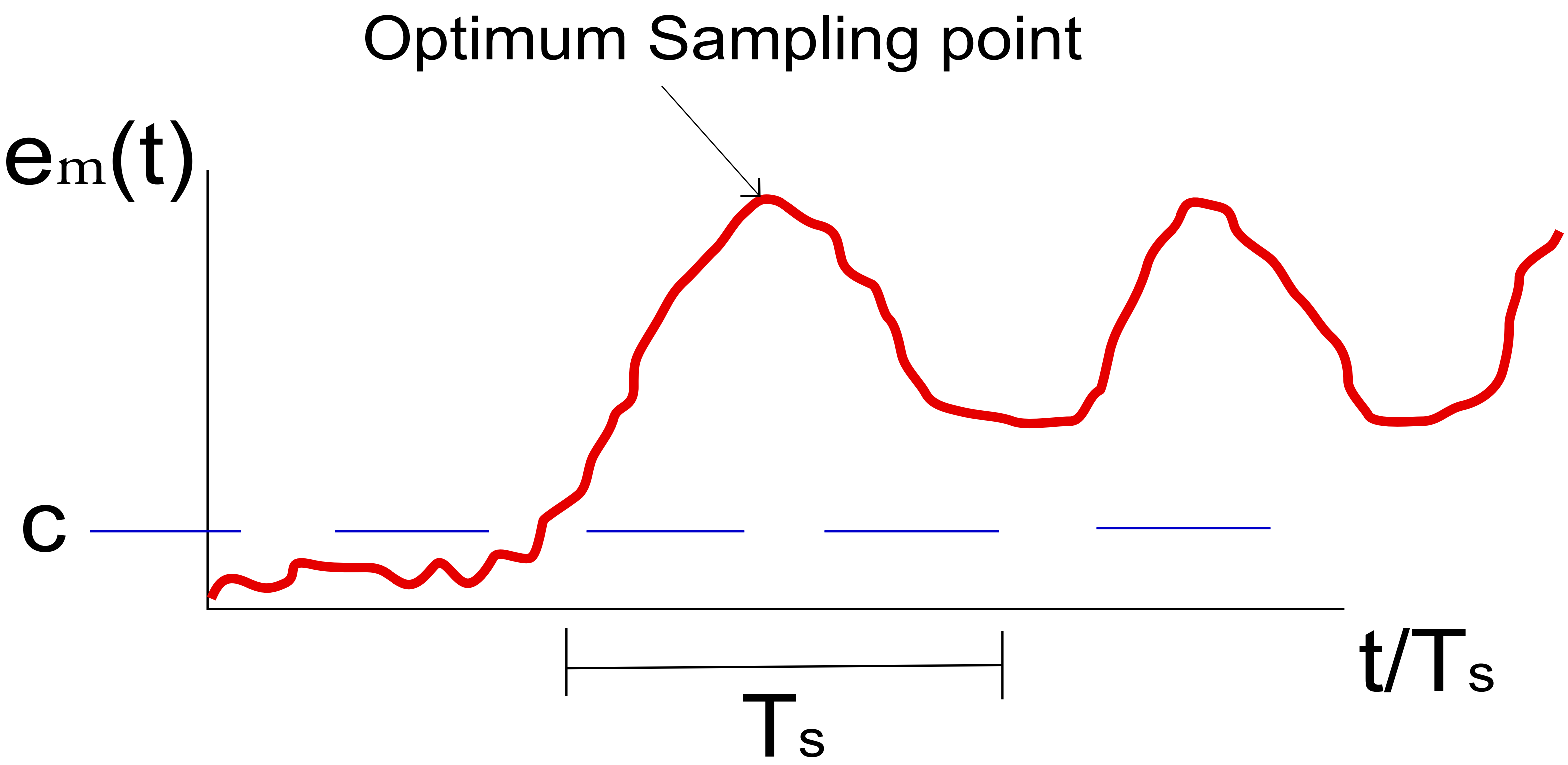
Estimation of ϵ is solved by selecting the sampling instance such that α is maximized. Recall the receiver structure



Synchronization

$$e_m(t) = \sqrt{m_I^2(t) + m_Q^2(t)}$$

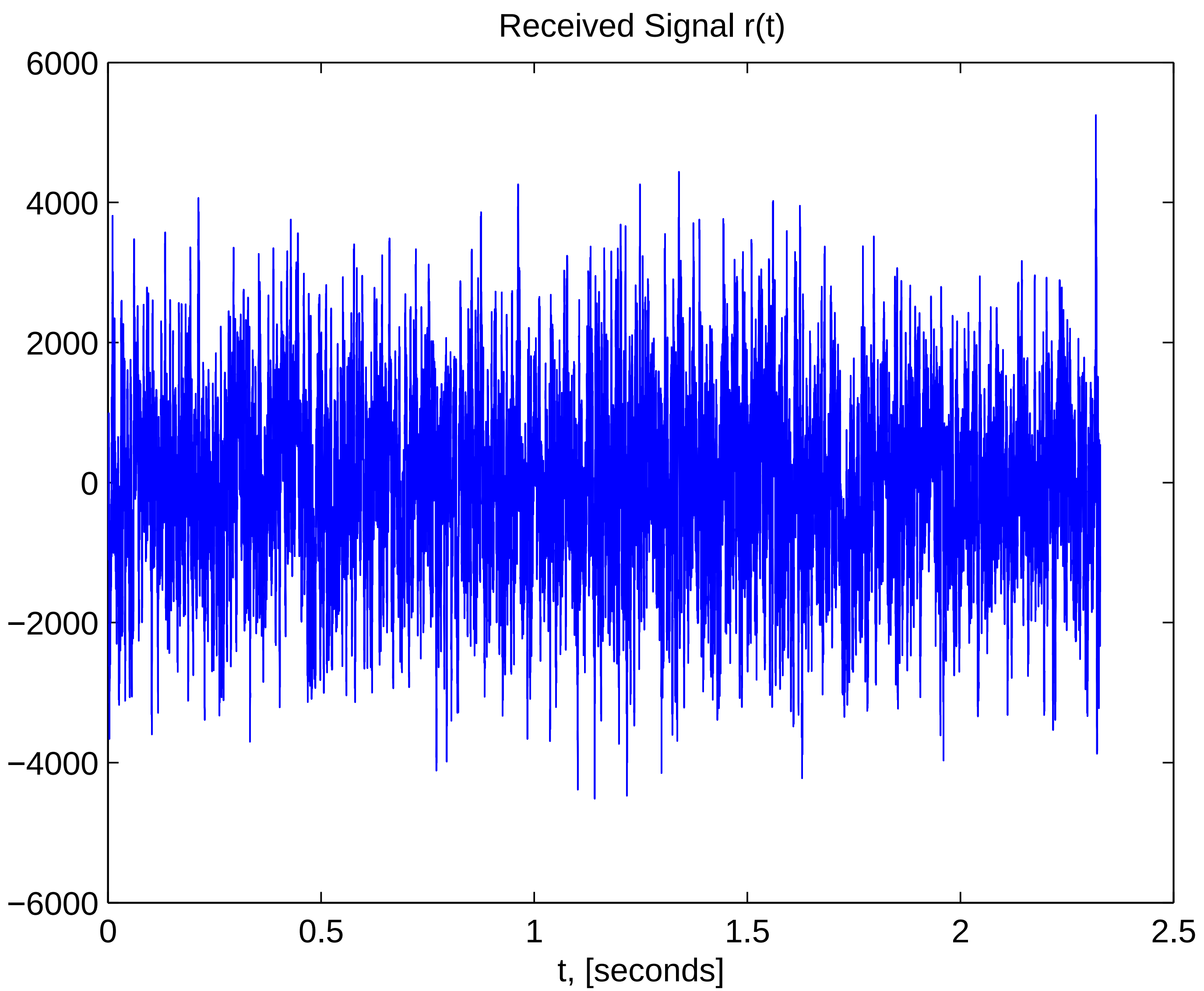
c is a threshold, easiest to find by trial and error



Example

The transmission starts and ends with pilots $2 + 2i$

Example


Received Signal r(t)
6000
4000
2000
0
−2000
−4000
−6000
0
0.5
1
1.5
2
2.5
t, [seconds]

Example

Construct $m_I(t)$ and $m_Q(t)$ as

$$m_I(t) = r(t)\cos(2\pi f_c t) \star p(t)$$

and

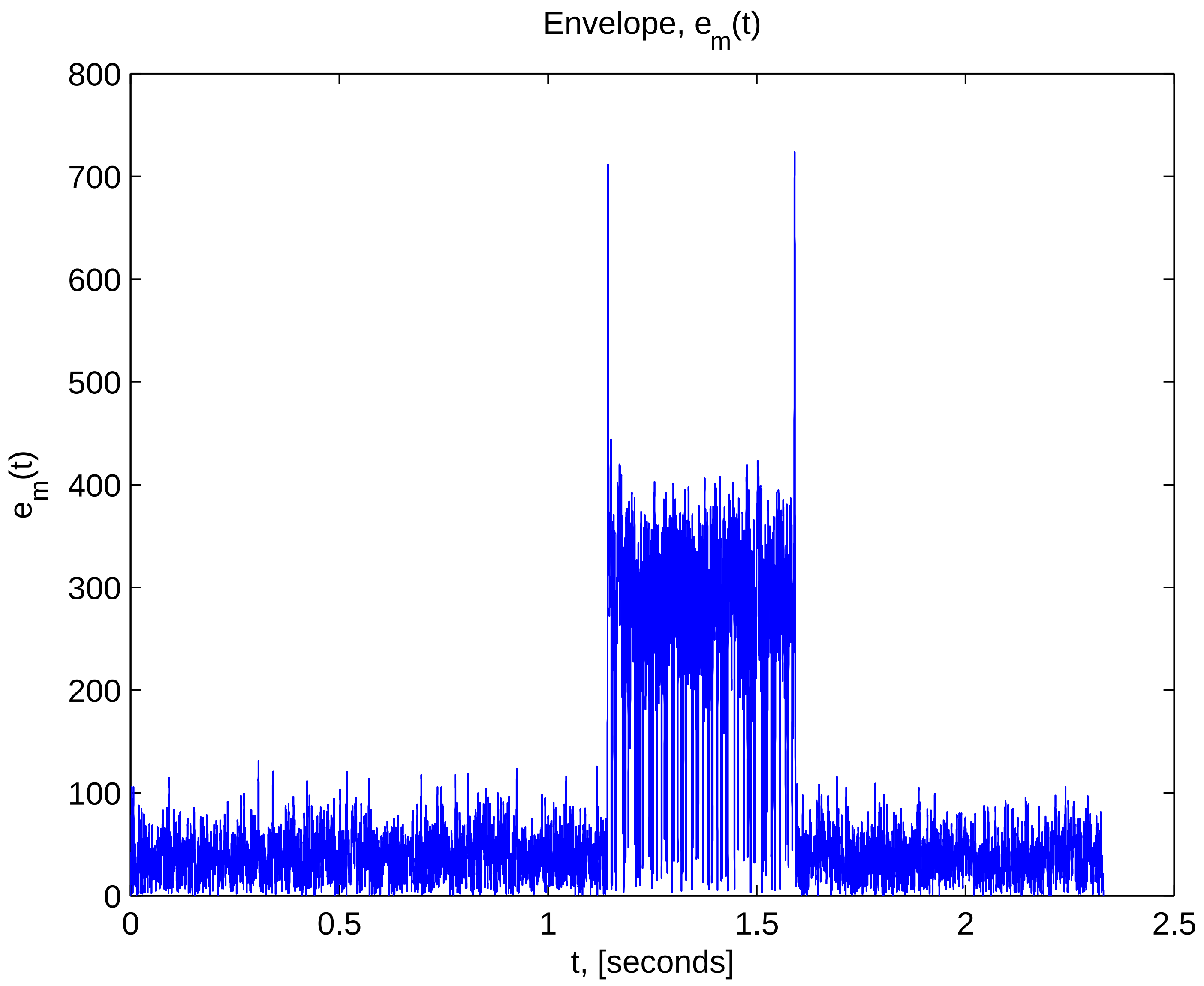
$$m_Q(t) = -r(t)\sin(2\pi f_c t) \star p(t)$$

Then generate

$$e_m(t) = \sqrt{m_I^2(t) + m_Q^2(t)}$$

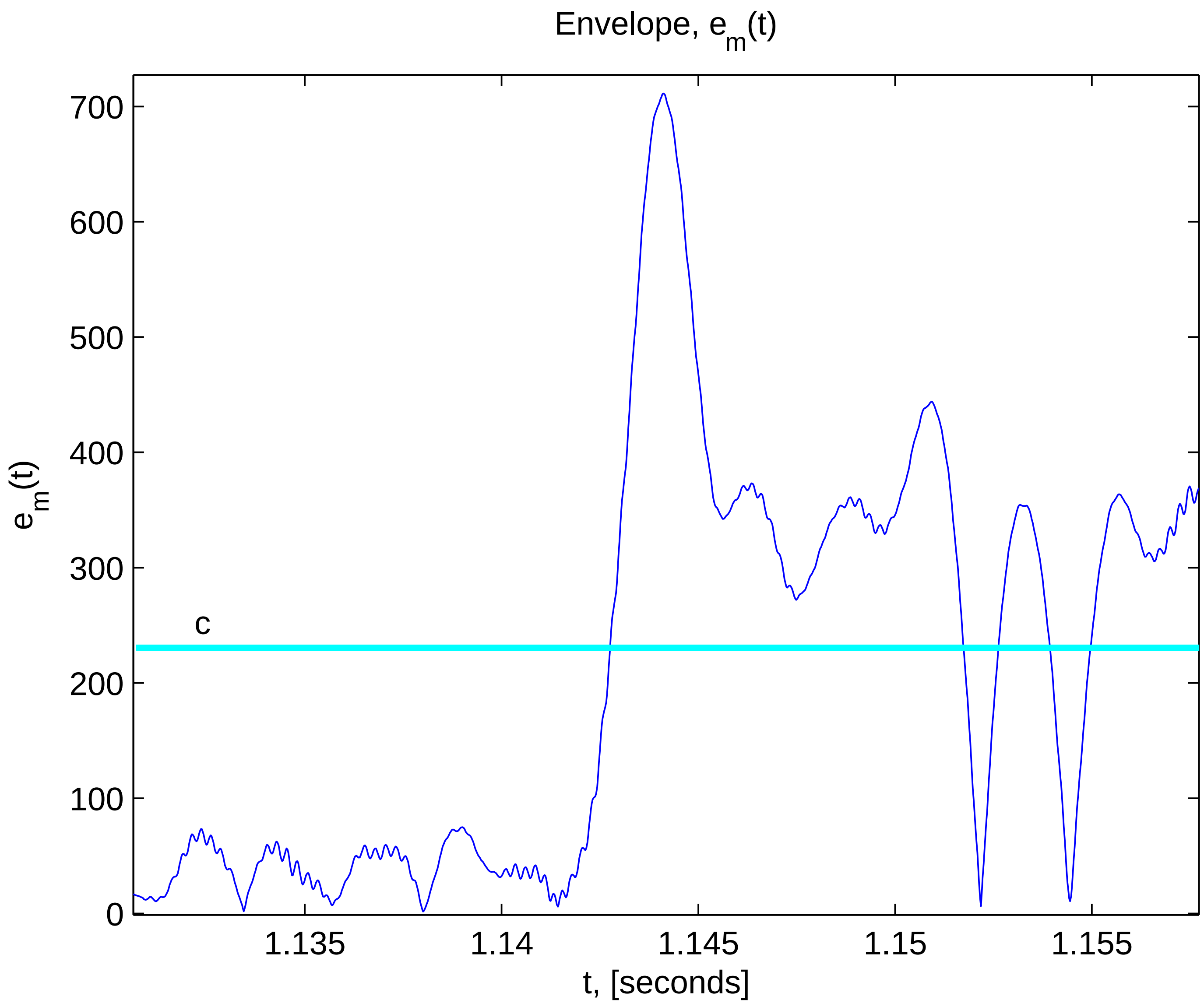
Plot

Example



Envelope, $e_m(t)$
800
700
600
500
400
300
200
100
0
$e_m(t)$
0
0.5
1
1.5
2
2.5
t, [seconds]

Example

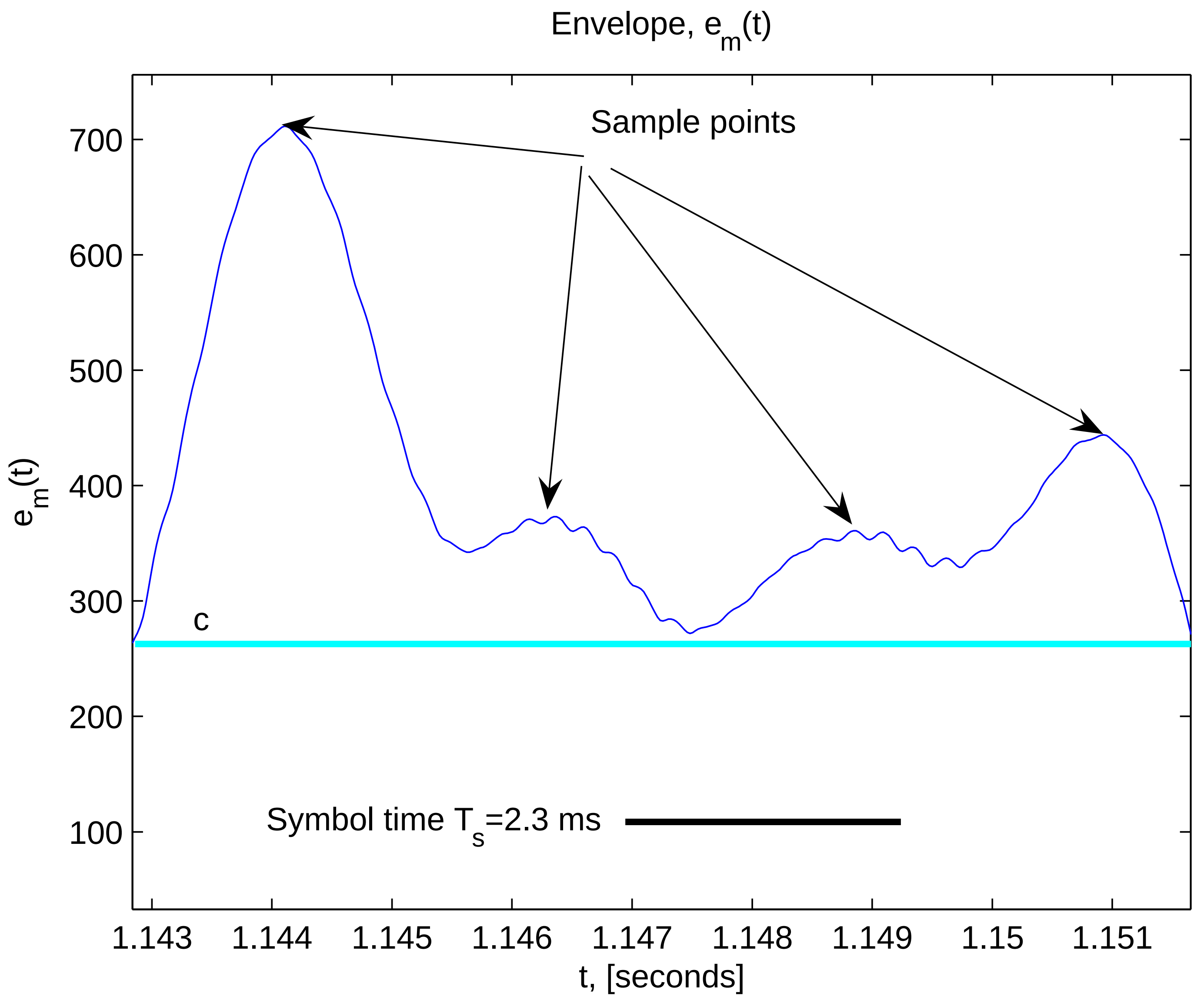


Envelope, $e_m(t)$
$e_m(t)$
700
600
500
400
300
200
100
0
c
1.135
1.14
1.145
1.15
1.155
t, [seconds]

Example

Optimal sampling point is at $T_{\text{Sample}} = 1.144$ seconds.

Example



Envelope, $e_m(t)$
Sample points
700
600
500
400
300
200
100
$e_m(t)$
c
Symbol time T_s=2.3 ms
1.143
1.144
1.145
1.146
1.147
1.148
1.149
1.15
1.151
t, [seconds]

Example

Sample at $T_{\text{Sample}} + kT_s$:

$$r[k] = m_I(T_{\text{Sample}} + kT_s) + im_Q(T_{\text{Sample}} + kT_s)$$

We get

$$r[k] = 4.38 - 5.60i \;\; 3.16 + 1.97i \;\; 2.79 + 2.16i \;\; 3.55 + 2.66i \;\; -2.73 - 2.25i....$$

Consequently

$$\alpha \exp(i\phi) = r[0]/(2+2i) = -0.30 - 2.49i$$

$$r[1]/\alpha \exp(i\phi) = -0.93 + 1.15i \;\; \text{and} \;\; r[2]/\alpha = -0.9899 + 0.9982i$$

So

$$\hat{a}[1] = -1 + i \;\; \text{and} \;\; \hat{a}[2] = -1 + i$$