LUND UNIVERSITY

EITG05 – Digital Communications

Week 6, Lecture 1

Receivers for bandpass signals
Equivalent baseband model
Compact description

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Week 6, Lecture 1

Equivalent baseband model of a communication system

Chapter 3.6: Receivers for Bandpass Signals

- 6.6.1 Homodyne reception

Chapter 3.7: A Compact Description

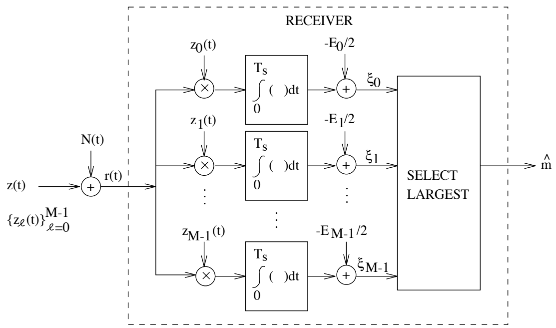
Pages 184 – 189 and 201 – 205

Exercises: 6.1a, 6.1c, 6.2, 6.3a, 6.4, 6.7a, 6.9a, 6.8

Laboratory takes place during weeks 6 and 7
each of you has to attend one lab session (4 hours)

General receiver for M-ary signaling

- Consider the general receiver structure from Chapter 4:

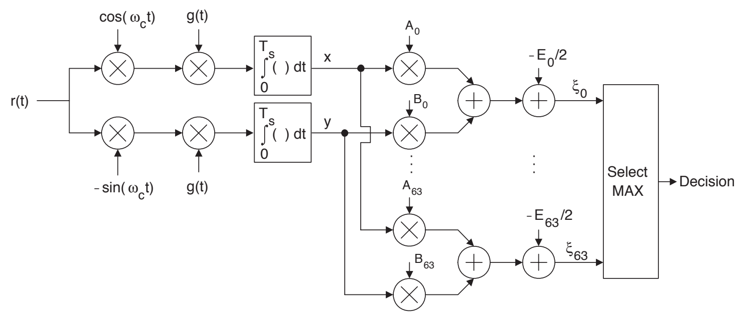


- Decision variables are computed by correlators or matched filters
- Each possible signal alternative is recreated in the receiver
- **Question:** can we apply this to bandpass signals? Yes!

But: recreating signals at large frequencies f_c is a challenge

Example: QAM Signaling

- Recall the simplified receiver considered in Example 4.4:

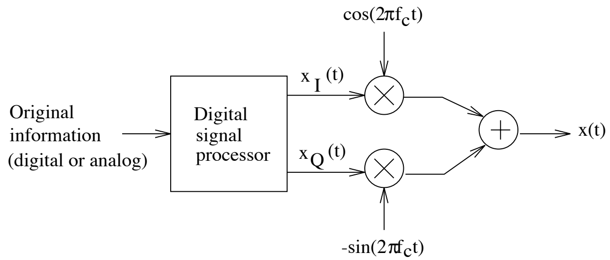


- Only two correlator branches are required instead of M
- Separation of carrier waveforms from baseband pulse possible

Our aim: a general baseband representation of the receiver

Transmission of bandpass signals

- Recall the transmitter structure from Chapter 3:

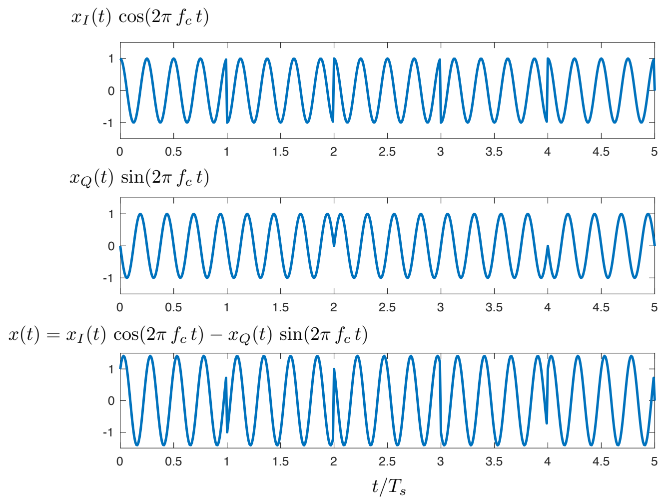


- A general bandpass signal can always be written as

$$x(t) = x_I(t)\,\cos(2\pi f_c t) \;-\; x_Q(t)\,\sin(2\pi f_c t)\;, \qquad -\infty \le t \le \infty$$

- $x_I(t)$: inphase component $\qquad x_Q(t)$: quadrature component

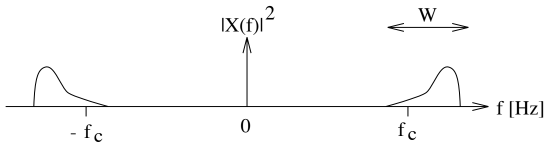
QPSK Example (considered in week 3)



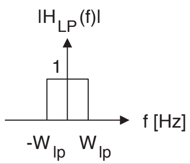
Exercise: determine $x_I(t)$ and $x_Q(t)$ from these figures

Receivers for bandpass signals

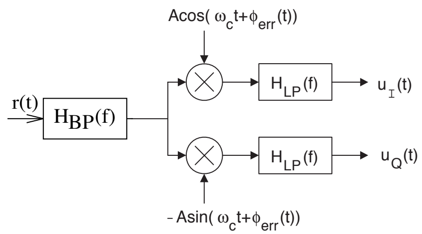
- **Our goal:** reproduce components $x_I(t)$ and $x_Q(t)$ at the receiver
- In the transmitted bandpass signal $x(t)$ these components were shifted to the carrier frequency f_c



- **Idea:** shifting the signal back to the baseband by multiplying with the carrier waveform again (recall Ex. 2.19 and Problem 3.9)
- A lowpass filter $H_{LP}(f)$ is then applied in the baseband to remove undesired other signals or copies from the carrier multiplication



Homodyne receiver frontend



- Receiver is not synchronized to transmitter: phase errors $\phi_{err}(t)$
- Assume first $r(t) = x_I(t)\,\cos(2\pi f_c t)$ $\quad$ ($x_Q(t) = 0$ and no noise)

$$u_I(t) = \big[x_I(t)\,\cos(2\pi f_c t)\cdot A\,\cos(2\pi f_c t + \phi_{err}(t))\big]_{LP}$$

$$= \Big[\frac{x_I(t)}{2} A\,\big(\cos(\phi_{err}(t)) + \cos(2\pi\,2 f_c t + \phi_{err}(t))\big)\Big]_{LP}$$

$$= \frac{x_I(t)}{2} A\,\cos(\phi_{err}(t))$$

- Likewise

$$u_Q(t) = -\frac{x_I(t)}{2} A\,\sin(\phi_{err}(t))$$

The impact of phase errors

- Assuming $r(t) = x_I(t)\cos(2\pi f_c t)$ we have found that

$$u_I(t) = \frac{x_I(t)}{2} A \cos(\phi_{err}(t)) , \quad u_Q(t) = -\frac{x_I(t)}{2} A \sin(\phi_{err}(t))$$

- **Ideal case:** $\phi_{err}(t) = 0$

$$u_I(t) = x_I(t)/2 \cdot A \quad \text{and} \quad u_Q(t) = 0$$

$\Rightarrow$ the inphase branch is independent of the quadrature branch

- **Phase errors:** $\phi_{err}(t) \neq 0$

$$u_I(t) < x_I(t)/2 \cdot A \quad \text{and} \quad u_Q(t) \neq 0 \ \text{(crosstalk)}$$

- If $\phi_{err}(t)$ changes randomly (jitter) the average $u_I(t)$ can vanish
- Ignoring the effect of phase errors can lead to bad performance

Question: what can we then do about phase errors?

Coherent receivers

- Assume now that we can estimate $\phi_{err}(t)$
- The signal $x_I(t)$ is contained in both $u_I(t)$ and $u_Q(t)$

$$u_I(t) = \frac{x_I(t)}{2} A \cos(\phi_{err}(t)) , \quad u_Q(t) = -\frac{x_I(t)}{2} A \sin(\phi_{err}(t))$$

- **Coherent reception:**

  by combining both components the signal can be recovered by

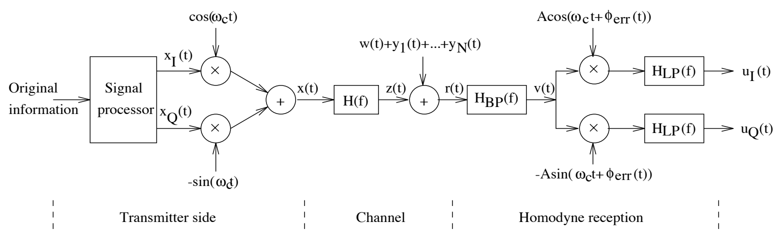
$$\hat{u}_I(t) = u_I(t) \cdot \cos(\phi_{err}(t)) - u_Q(t) \cdot \sin(\phi_{err}(t))$$

$$= \frac{x_I(t)}{2} A \cos^2(\phi_{err}(t)) + \frac{x_I(t)}{2} A \sin^2(\phi_{err}(t)) = \frac{x_I(t)}{2} A$$

- Observe: same result as in the ideal case $\phi_{err}(t) = 0$

Compare: non-coherent DPSK receiver (last week, p. 400-403) can be used if phase estimation is not possible

Overall transmission model



- The signal $y(t)$ is given by

$$y(t) = z(t) + w(t) = x(t) \ * \ h(t) + w(t)$$

- It can be written as

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

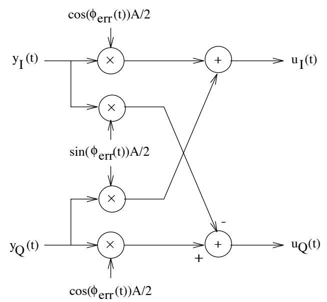
Can we express $u_I(t)$ and $u_Q(t)$ in terms of $x_I(t)$ and $x_Q(t)$?

Inphase and quadrature relationship

- With the complete signal $r(t)$ entering the receiver the output signals become

$$u_I(t) = \left[y(t) A \cos(2\pi f_c t + \phi_{err}(t)) \right]_{LP}$$
$$= \frac{y_I(t)}{2} A \cos(\phi_{err}(t))$$
$$+ \frac{y_Q(t)}{2} A \sin(\phi_{err}(t))$$

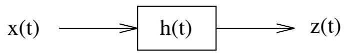
$$u_Q(t) = \left[-y(t) A \sin(2\pi f_c t + \phi_{err}(t)) \right]_{LP}$$
$$= \frac{y_Q(t)}{2} A \cos(\phi_{err}(t))$$
$$- \frac{y_I(t)}{2} A \sin(\phi_{err}(t))$$



Including the channel filter

- Before we can relate $y(t) = z(t) + w(t)$ to $x(t)$ we need to consider the effect of the channel

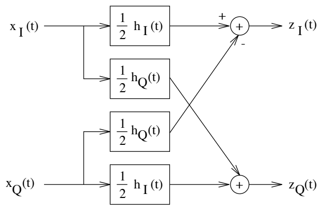
$$z(t) = x(t) * h(t)$$



- We assume that the impulse response $h(t)$ can be represented as a bandpass signal

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t)$$

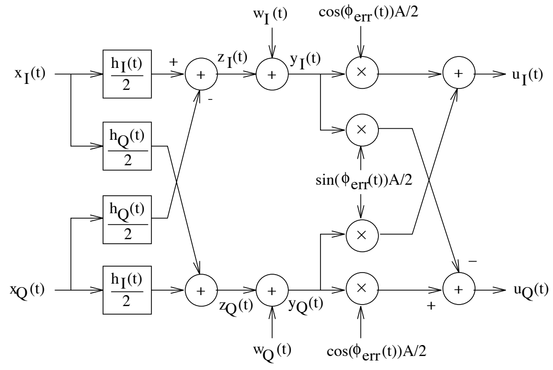
- With some calculations the signals can be written as (p. 159-160)



$$z_I(t) = \frac{1}{2}\big(x_I(t) * h_I(t) - x_Q(t) * h_Q(t)\big)$$
$$z_Q(t) = \frac{1}{2}\big(x_I(t) * h_Q(t) + x_Q(t) * h_I(t)\big)$$

Equivalent baseband model

- Combining the channel with the receiver frontend we obtain



- Observe that all the involved signals are in the baseband
- The same is true for channel filter, noise and phase error

Digital signal processing can be applied easily in baseband

What happened with the carrier waveforms?

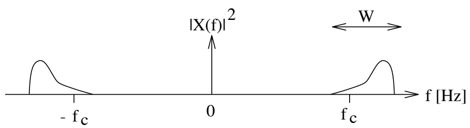
A compact description

- A more compact description is possible by combining $x_I(t)$ and $x_Q(t)$ to an equivalent baseband signal

$$\tilde{x}(t) = x_I(t) + j x_Q(t)$$

- The transmitted signal can then be described as

$$x(t) = Re\left\{(x_I(t) + j x_Q(t))\, e^{+j2\pi f_c t}\right\} = Re\left\{\tilde{x}(t)\, e^{+j2\pi f_c t}\right\}$$



- With $Re\{a\} = (a + a^*)/2$ we can write

$$x(t) = \frac{\tilde{x}(t)}{2} \cdot e^{+j2\pi f_c t} + \frac{\tilde{x}^*(t)}{2} \cdot e^{-j2\pi f_c t}$$

A compact description

- Let us first ignore the effect of the channel: $w(t) = 0,\ h(t) = \delta(t)$
- The receiver can invert the frequency shift operation by

$$\tilde{u}(t) = \left[x(t) \cdot A\, e^{-j(2\pi f_c t + \phi_{err}(t))}\right]_{LP}$$

- Using the expression for $x(t)$ from the previous slide we get

$$\tilde{u}(t) = \left[\frac{A}{2}\left(\tilde{x}(t) e^{+j2\pi f_c t} + \tilde{x}^*(t) e^{-j2\pi f_c t}\right) \cdot e^{-j(2\pi f_c t + \phi_{err}(t))}\right]_{LP}$$
$$= \frac{\tilde{x}(t)}{2} A \cdot e^{-j\phi_{err}(t)} = u_I(t) + j u_Q(t)$$

- Observe that this expression is equivalent to our earlier result

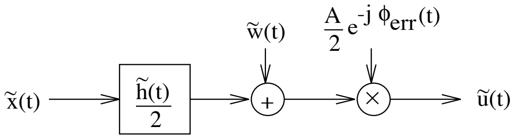
$$\tilde{u}(t) = \left(\frac{x_I(t)}{2} A\cos(\phi_{err}(t)) + \frac{x_Q(t)}{2} A\sin(\phi_{err}(t))\right)$$
$$+ j\left(\frac{x_Q(t)}{2} A\cos(\phi_{err}(t)) - \frac{x_I(t)}{2} A\sin(\phi_{err}(t))\right)$$

Compact equivalent baseband model

- The effect of the channel filter becomes

$$\tilde{z}(t) = z_I(t) + j z_Q(t) = \tilde{x}(t) * \frac{\tilde{h}(t)}{2}$$

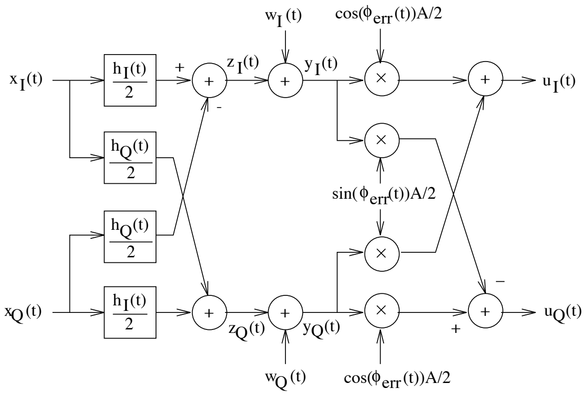
- Combining these parts and the noise we obtain the simple model

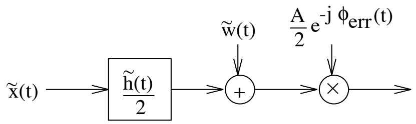


$$\tilde{u}(t) = \left[\left(\tilde{x}(t) * \frac{\tilde{h}(t)}{2}\right) + \tilde{w}(t)\right] \cdot e^{-j\phi_{err}(t)} \cdot \frac{A}{2} \,, \quad \tilde{w}(t) = w_I(t) + j w_Q(t)$$

- Complex signal notation simplifies expressions significantly

The two equivalent baseband models





M-ary QAM signaling

- Considering M-ary QAM signals we get

$$x_I(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\, g(t - nT_s) \,, \quad x_Q(t) = \sum_{n=-\infty}^{\infty} B_{m[n]}\, g(t - nT_s)$$

- Let us now introduce

$$\tilde{A}_m[n] = A_{m[n]} + j B_{m[n]}$$

- Then our complex baseband signal $\tilde{x}(t)$ can be written as

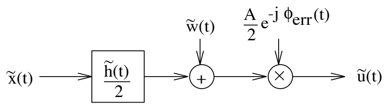
$$\tilde{x}(t) = x_I(t) + j\, x_Q(t) = \sum_{n=-\infty}^{\infty} \tilde{A}_{m[n]}\, g(t - nT_s)$$

- **Example:** (on the board)
  Consider 4-QAM transmission of $\mathbf{b} = 1\,0\,1\,1\,1\,0\,0\,1$
  Determine $A_{m[n]}$, $B_{m[n]}$ and $\tilde{A}_{m[n]}$

How can we design the receiver for QAM signals?

Matched filter receiver

- At the receiver we see the complex baseband signal $\tilde{u}(t)$

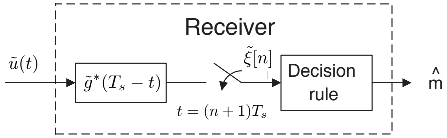


- If we know the channel we can design a matched filter for

$$\tilde{z}(t) = \tilde{x}(t) * \frac{\tilde{h}(t)}{2} \quad \Rightarrow \quad \tilde{v}(t) = \tilde{z}^*(T_s - t)$$

- It is often convenient to match $\tilde{v}(t)$ to the pulse $g(t)$ instead

$$\tilde{v}(t) = g^*(T_s - t) \quad \Rightarrow \quad \tilde{\xi}[n] = \left[\tilde{u}(t) * g^*(T_s - t)\right]_{t=(n+1)T_s}$$



Decision rule

- Consider now $\tilde{h}(t) = \delta(t)$ and $\tilde{w}(t) = 0$
- The ideal values of the decision variable are then given by

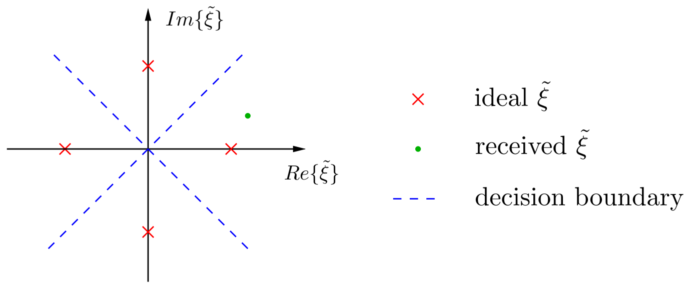
$$\begin{aligned}\tilde{\xi}_{m[n]} &= \left[\tilde{u}(t) \, * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\ &= \left[\left(\tilde{A}_{m[n]} g(t-nT_s) \cdot e^{-j\phi_{err}(t)} \cdot \frac{A}{2}\right) * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\ &= \tilde{A}_{m[n]} e^{-j\phi_{err}(t)} \cdot \frac{A}{2} \left[g(t-nT_s) \, * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\ &= \tilde{A}_{m[n]} e^{-j\phi_{err}((n+1)T_s)} \cdot \frac{A}{2} \, E_g\end{aligned}$$

- Due to noise $w(t) \neq 0$ and non-ideal channel $\tilde{h}(t)$ the decision variables at the receiver will differ from these ideal values
- The Euclidean distance receiver will base its decision on the ideal value $\tilde{\xi}_{m[n]}$ which is closest to the received value $\tilde{\xi}[i]$

Example: 4-PSK

- Assuming $\phi_{err}(t) = 0$ we obtain the ideal decision variables

$$\tilde{\xi}_{m[n]} = \tilde{A}_{m[n]} \cdot \frac{A}{2} \, E_g = (A_{m[n]} + jB_{m[n]}) \cdot \frac{A}{2} \, E_g$$

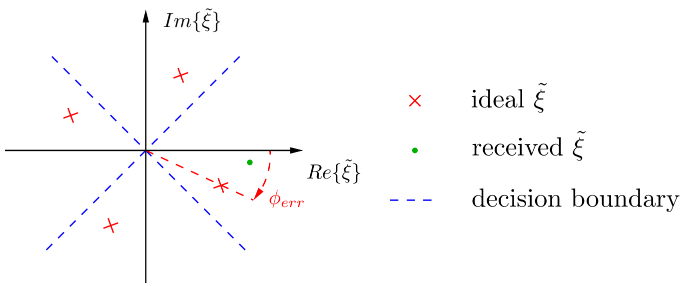


- Based on the received value $\tilde{\xi}[n]$ we decide for

$$\hat{m}[n]: \quad \tilde{A}_{\hat{m}[n]} = (1 + j \cdot 0)$$

Example: 4-PSK with phase offset

- Consider now a constant phase offset of $\phi_{err}(t) = \phi_{err} = 25°$
- As a result the values $\tilde{\xi}_{m[n]}$ and $\tilde{\xi}[n]$ are rotated accordingly



How can we compensate for ϕ_{err}?

1. we can rotate the decision boundaries by the same amount
2. or we can rotate back $\tilde{\xi}[n]$ by multiplying with $e^{+j\phi_{err}}$

Summary: M-ary QAM transmission

- We can describe the transmitted messages $\tilde{A}_{\hat{m}[n]}$ and the decision variables $\tilde{\xi}[n]$ at the receiver as complex variables
- The effect of the noise $\tilde{w}(t)$ and the channel filter $\tilde{h}(t)$ on $\tilde{\xi}[n]$ can be described by the equivalent baseband model
- The transmitter and receiver frontends can be separated from the (digital) baseband processing
- Assumptions:
  - the pulse shape $g(t)$ satisfies the ISI-free condition
  - the carrier frequency f_c is much larger than the bandwidth of $g(t)$
- Under these conditions the design of the baseband receiver and its error probability analysis can be applied as in Chapter 4