EITG05 – Digital Communications

Week 5, Lecture 1

Gap to Capacity
Multiuser Receiver, Non-coherent Receiver
Intersymbol Interference

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Week 5, Lecture 1

Chapter 5: Receivers in Digital Communication Systems – Part II

- Fig. 5.17: gap to capacity
- 5.4.3 A simplified model of multiuser communication
- 5.4.5 Differential phase-shift-keying

Chapter 6: Intersymbol Interference

- 6.1 Increasing the signaling rate – ISI

Pages 369, 395 – 396, 400 – 403, and 435 – 446

Exercises: 4.19, 4.21, Example 4.19 on page 279, 4.13, 4.12, Example 4.4 on page 242, 4.18

How can we achieve large data rates?

- The bit rate R_b can be increased in different ways
- We can select a signal constellation with large M
  $\Rightarrow$ this typically increases the error probability P_s
  exception: orthogonal signals (FSK): require more bandwidth W
- Achieving equal P_s with larger M is possible by increasing $\mathcal{E}_b/N_0$
  $\Rightarrow$ this reduces the energy efficiency
- We can also increase R_b by increasing the bandwidth W
  $\Rightarrow$ this does not improve the bandwidth efficiency $\rho = R_b/W$

Question:
what is the largest achievable rate R_b for a given error probability P_s, channel quality $\mathcal{E}_b/N_0$ and bandwidth W?

This question was answered by Claude Shannon in 1948:
"A mathematical theory of communication"
Course EITN45: Information Theory (VT2)

A fundamental limit: channel capacity

- Consider a single-path channel ($|H(f)|^2 = \alpha^2$) with finite bandwidth W and additive white Gaussian noise (AWGN) $N(t)$
- The capacity for this channel is given by

$$\mathcal{C} = W \log_2 \left(1 + \frac{\mathcal{P}_z}{N_0 W}\right) \text{ [bps]}$$

- Shannon showed that reliable communication requires that

$$R_b \leq \mathcal{C}$$

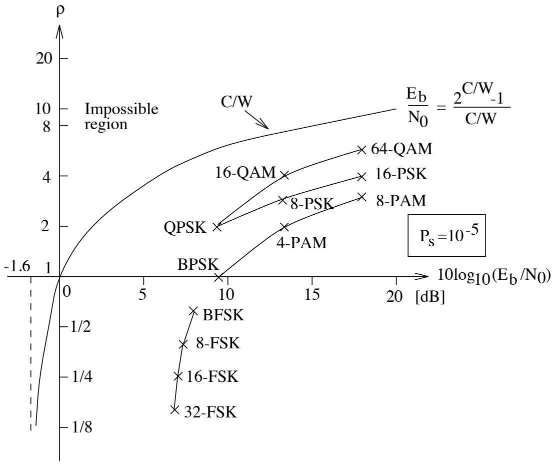
- **Observe:** the capacity formula does not include P_s (why?)
- Shannon also showed that if $R_b < \mathcal{C}$, then the probability of error P_s can be made arbitrarily small

$$P_s \rightarrow 0$$

  if messages are coded in blocks of length $N \rightarrow \infty$

Bandwidth efficiency and gap to capacity

(p. 369)



- $\rho \le C/W$: reliable communication is impossible above
- this limit can be approached with channel coding

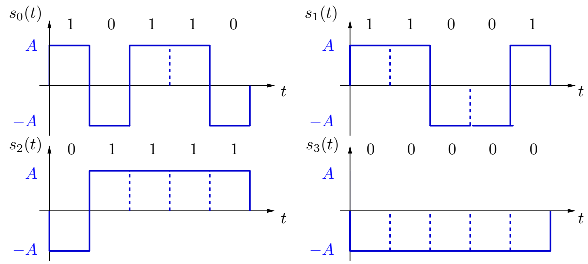
How does channel coding work?

- We have seen that a large minimum distance d^2_{min} between signals is required to improve the energy efficiency
- For binary signaling ($M = 2$) we have seen that $d^2_{min} \le 2$

Idea of coding:

- generate M binary sequences of length N
- use binary antipodal signaling to create M signals $s_\ell(t)$

Example: $N = 5$, $M = 4$, $g_{rec}(t)$ pulse with $T = T_s/N$ (what is D^2_{min}?)



Increasing d^2_{min} with coding

- In our example we have

$$D^2_{min} = 4A^2\,T \cdot 3 = 4\,E_g\,3 = 12\,E_g$$

- Normalizing by the average energy $\mathcal{E}_b = N\,E_g/k$ this gives

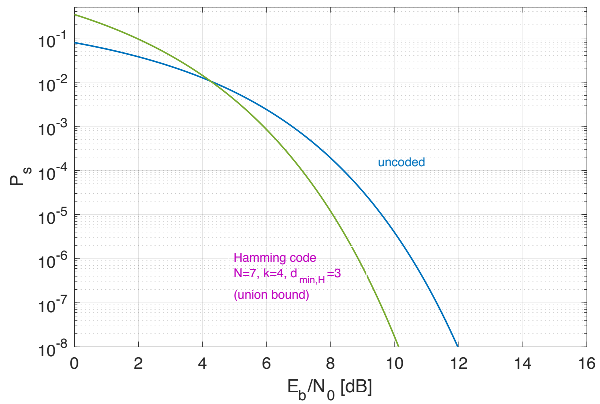
$$d^2_{min} = \frac{D^2_{min}}{2\mathcal{E}_b} = \frac{12\,E_g}{2\,N/k\,E_g} = 6 \cdot \frac{k}{N} = \frac{12}{5} = 2.4$$

- Let $d_{min,H}$ denote the minimum Hamming distance between the binary code sequences $\Rightarrow$ in our example: $d_{min,H} = 3$
- Then we can write

$$d^2_{min} = 2\,\frac{k}{N}\,d_{min,H} = 2\,R\,d_{min,H}\ ,$$

where $R = k/N$ is called the code rate
- Larger $d_{min,H}$ values can be achieved with larger N

Example: symbol error probability

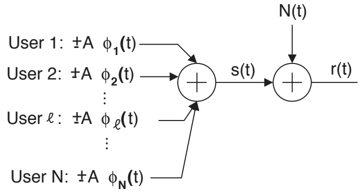


- Hamming code, $N = 7$, $k = 4$, $d_{min,H} = 3 \Rightarrow d^2_{min} = 3.43$
- How can we construct good codes?

EITN70: Channel Coding for Reliable Communication (HT2)

Multiuser Communication

(p. 395/396)



A simple model:

- N users transmit at same time with orthonormal waveforms $\phi_\ell(t)$
- Binary antipodal signaling is used in this example, such that

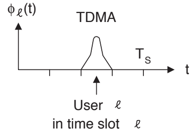
$$s(t) = \sum_{n=1}^{N} A_n\, \phi_n(t)\ , \quad A_n \in \pm A$$

- The orthonormal waveforms satisfy

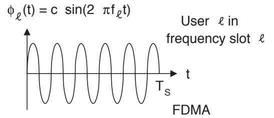
$$\int_0^{T_s} \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 0 & \text{if } i \neq j\ , \\ 1 & \text{if } i = j \end{cases}$$

Multiuser Communication

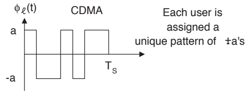
- The separation of users can be achieved in different ways
- **TDMA:** (time-division multiple access)



- **FDMA / OFDMA:** (frequency-division multiple access)

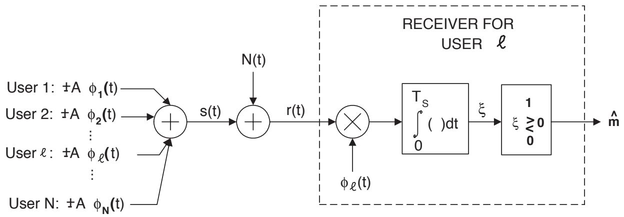


- **CDMA:** (code-division multiple access)



- **MC-CDMA:** (multi-carrier CDMA) combined OFDM/CDMA

Receiver for Multiuser Communication



- This permits a simple receiver structure for each user ℓ
- The decision variable becomes

$$\xi = \int_0^{T_s} \phi_\ell(t)\, r(t)\, dt = \int_0^{T_s} \phi_\ell(t) \left(\sum_{n=1}^{N} A_n\, \phi_n(t) + N(t) \right) dt$$

$$= A_\ell + \int_0^{T_s} \phi_\ell(t)\, N(t)\, dt = A_\ell + \mathcal{N}$$

$\Rightarrow$ receiver is only disturbed by noise and not by other users!

Non-coherent receivers

- With phase-shift keying (PSK) the message $m[n]$ at time nT_s is put into the phase θ_n of the transmit signal

$$s(t) = g(t)\, \sqrt{2E}\, \cos(2\pi f_c\, t + \theta_n)\ , \quad nT_s \leq t \leq (n+1)T_s$$

- The channel introduces some attenuation α, some additive noise $N(t)$ and also some phase offset ν into the received signal

$$r(t) = \alpha\, g(t)\, \sqrt{2E}\, \cos(2\pi f_c\, t + \theta_n + \nu) + N(t)$$

- **Challenge:** the optimal receiver needs to know α and ν
- In some applications an accurate estimation of ν is infeasible (cost, complexity, size)
- **Non-coherent receivers:**
  receiver structures that can work well without knowledge of the exact phase offset

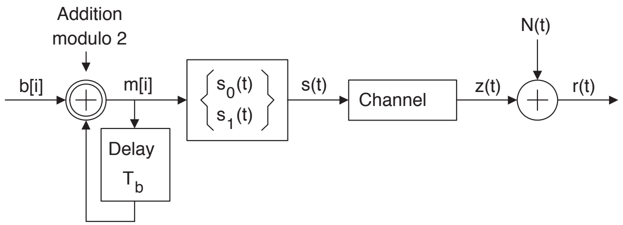
How can we modify our PSK transmission accordingly?

Differential Phase Shift Keying

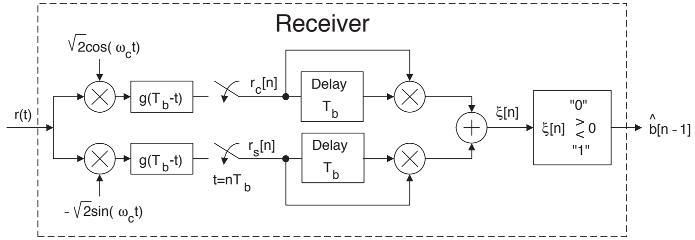
- With differential PSK, the message $m[n] = m_\ell$ is mapped to the phase according to

$$\theta_n = \theta_{n-1} + \frac{2\pi\ell}{M} \qquad \ell = 0, \ldots, M-1$$

- The transmitted phase θ_n depends on both θ_{n-1} and $m[n]$
- This differential encoding introduces memory and the transmitted signal alternatives become dependent
- **Example 5.25:** binary DPSK



Differential Phase Shift Keying ($M = 2$)



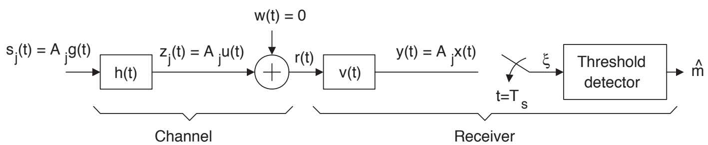
- The receiver uses no phase offset ν in the carrier waveforms
- Without noise, the decision variable is

$$\begin{aligned}\xi[n] &= r_c[n]\, r_c[n-1] + r_s[n]\, r_s[n-1] \\ &= A\cos(\theta_{n-1}+\nu)\, A\cos(\theta_{n-2}+\nu) + A\sin(\theta_{n-1}+\nu)\, A\sin(\theta_{n-2}+\nu) \\ &= A^2\cos(\theta_{n-1}-\theta_{n-2}) \quad \Rightarrow \text{ independent of } \nu\end{aligned}$$

- Note: non-coherent reception increases variance of noise

Intersymbol Interference (ISI)

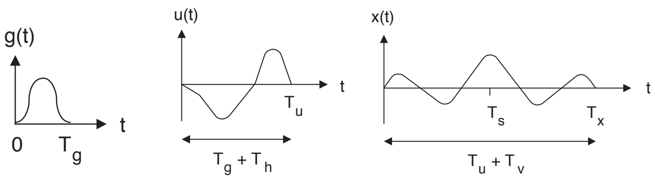
- Consider transmission of a single M-ary PAM signal alternative



- In the noise-free case ($w(t) = 0$) the signal $x(t)$ can be written as

$$x(t) = u(t) \ast v(t) = g(t) \ast h(t) \ast v(t)$$

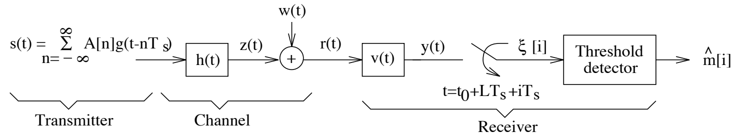
Example:



What happens if $T_u = T_g + T_h \geq T_s$? $\Rightarrow$ ISI occurs

Intersymbol Interference (ISI)

- For $R_s = 1/T_s < 1/T_u$ we can use the ML receiver from Chapter 4
- **Question:** can we use such a receiver for larger rates $R_s \geq 1/T_u$?
- Consider the following receiver structure (compare to last slide)



- Note that $z(t)$ now is a superposition of overlapping pulses $u(t)$
- The signal $y(t)$ after the receiver filter $v(t)$ is

$$y(t) = \sum_{n=-\infty}^{\infty} A[n]\, x(t - nT_s) + w_c(t) ,$$

where $w_c(t)$ is a filtered Gaussian process
- The decision variable is obtained after sampling

$$\xi[i] = y(\mathcal{T} + iT_s) , \quad \mathcal{T} = t_0 + LT_s , \text{ where } LT_s \leq T_u$$

Discrete time model for ISI

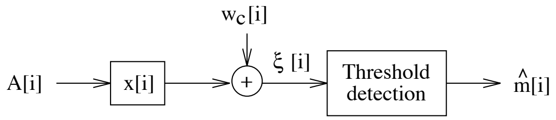
- According to our model the decision variable can be written as

$$\xi[i] = y(\mathcal{T} + iT_s) = \sum_{n=-\infty}^{\infty} A[n]\, x(\mathcal{T} + iT_s - nT_s) + w_c(\mathcal{T} + iT_s)$$

- Let us introduce the discrete sequences

$$x[i] = x(\mathcal{T} + iT_s)\ , \quad w_c[i] = w_c(\mathcal{T} + iT_s)$$

- This leads to the following discrete-time model of our system

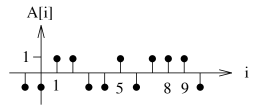


$$\xi[i] = \sum_{n=-\infty}^{\infty} A[n]\, x[i-n] + w_c[i] = A[i] * x[i] + w_c[i]$$

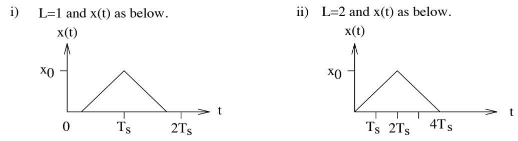
Remark: the discrete-time impulse response $x[i]$ represents pulse shape $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$

Example 6.1

The transmitted sequence of amplitudes $A[i]$ is given as,



Calculate, and plot, the sequence of decision variables $\xi[i]$ in Figure 6.2, for $0 \le i \le 8$, in the noiseless case (i.e. $w(t) = 0$) if $t_0 = 0$ and if the output pulse $x(t)$ is:



- i) $\xi[i] = x_0 A[i]$ ii) $\xi[i] = \frac{x_0}{2} A[i+1] + x_0 A[i] + \frac{x_0}{2} A[i-1]$