LUND UNIVERSITY

EITG05 – Digital Communications

Week 3, Lecture 2

Carrier Modulation Techniques
Receivers in Digital Communication Systems

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Thursday, September 14, 2017

Week 3, Lecture 2

Chapter 3: Information Transmission
with Carrier Modulation Techniques

- 3.4 Bandpass filtering
  - 3.4.3 N-ray channel model
- 3.5 Interference and noise
  - 3.5.3 Noise

Chapter 4: Receivers in Digital Communication Systems – Part I

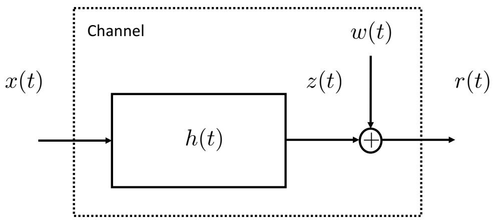
- 4.1 Introduction
- 4.2 Basic concepts and principles
- 4.3 The minimum Euclidean distance receiver

Pages 167 – 184 (excluding 3.5.1 - 3.5.2) and 227 – 244

Exercises: 3.5, 3.6, Example 3.7 on page 135,
3.9, 3.10b, 3.19, 3.7, 3.22

The Channel

- The channel is often modeled as time-invariant filter with noise



- $h(t)$ is the channel impulse response and $w(t)$ the additive noise
- The received signal becomes

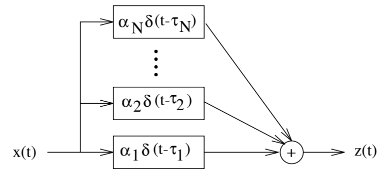
$$r(t) = x(t) * h(t) + w(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau + w(t)$$

- The simplest case is an attenuated noisy channel:

$$h(t) = \alpha\, \delta(t) \qquad \Rightarrow r(t) = \alpha\, s(t) + w(t)$$

N-ray Channel Model

- In many applications (wired and wireless) the transmitted signal $x(t)$ reaches the receiver along several different paths
- Such multi-path propagation motivates the N-ray channel model



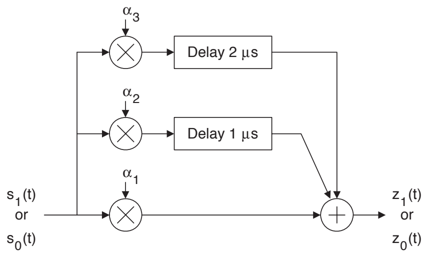
- The output signal becomes

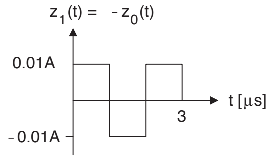
$$z(t) = \sum_{i=1}^{N} \alpha_i\, x(t-\tau_i) = x(t) * h(t)$$

- The impulse response $h(t)$ and its Fourier transform are given by

$$h(t) = \sum_{i=1}^{N} \alpha_i\, \delta(t-\tau_i)\ , \qquad H(f) = \sum_{i=1}^{N} \alpha_i\, e^{-j2\pi f \tau_i}$$

Example 3.19: multipath propagation





$$s_1(t) = -s_0(t) = \begin{cases} A & , \quad 0 \le t \le 10^{-6} \\ 0 & , \quad otherwise \end{cases}$$

$\alpha_1 = 0.01, \alpha_2 = -0.01, \alpha_3 = 0.01$

- The channel (= filter) increases the length of the signals
- Signals exceed their time interval and will overlap if T_s is not increased accordingly ⇒ inter-symbol interference (ISI)

Example 3.20

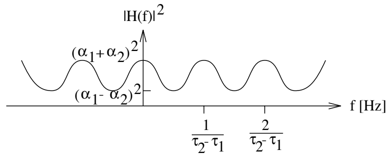
EXAMPLE 3.20

Calculate and sketch $|H(f)|^2$ for the 2-ray channel model.

Solution:

From (3.128) we obtain,

$$\begin{aligned} H(f) &= \alpha_1 e^{-j2\pi f \tau_1} + \alpha_2 e^{-j2\pi f \tau_2} = \\ &= e^{-j2\pi f \tau_1}\left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2-\tau_1)}\right) \\ |H(f)|^2 &= \left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2-\tau_1)}\right)\left(\alpha_1 + \alpha_2 e^{+j2\pi f(\tau_2-\tau_1)}\right) = \\ &= \alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2\left(e^{j2\pi f(\tau_2-\tau_1)} + e^{-j2\pi f(\tau_2-\tau_1)}\right) = \\ &= \alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cos(2\pi f(\tau_2-\tau_1)) \end{aligned}$$



Channel fading: some frequencies are attenuated strongly

Features of Multipath Channels

Challenges:

- the receiver needs to know the channel
- training sequences need be transmitted for channel estimation
- the impulse response can change over time
- the line-of-sight (LOS) component is sometimes not received

Opportunities:

- with multiple paths we can collect more signal energy
- receiver can work without direct LOS component
- channel knowledge, once we have it, can give useful information: Examples: distance, angle of arrival, speed (Doppler)
- positioning/navigation is often based on channel estimation

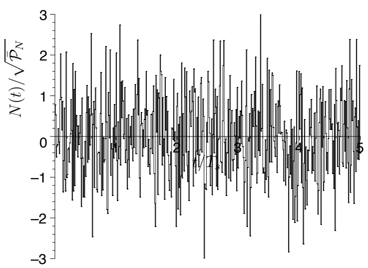
If you want to know more:

EITN85: Wireless Communication Channels, VT 1

Channel Noise

- In almost all applications the received signal $r(t)$ is disturbed by some additive noise $N(t)$:

$$r(t) = z(t) + N(t)$$



- Since the received noise disturbs that transmitted signal, we need to characterize its influence on the performance in terms of bit error rate or achievable information bit rate

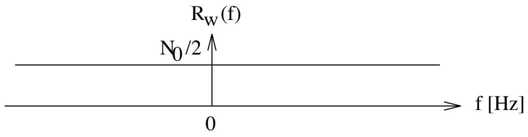
White Gaussian Noise

- White Gaussian noise $w(t)$ is a common model for background noise, such as created by electronic equipment
- The samples of $w(t)$ have a zero-mean Gaussian distribution
- Any two distinct samples of $w(t)$ are uncorrelated

$$r_w(\tau) = E\{w(t+\tau)\,w(t)\} = \frac{N_0}{2}\,\delta(\tau)$$

- This leads to a constant power spectral density

$$R_w(f) = \int_{-\infty}^{\infty} r_w(\tau)\,e^{-j2\pi f\tau}\,d\tau = \frac{N_0}{2}\,, \quad -\infty \leq f \leq \infty$$



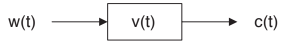
All frequencies are disturbed equally strongly

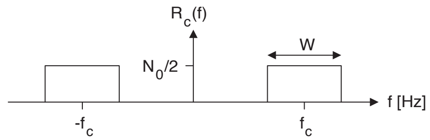
Filtered Gaussian Noise

- In reality we usually deal with filtered noise of limited bandwidth, so-called colored noise
- Assuming that white Gaussian noise $w(t)$ passes a filter $v(t)$ we obtain colored noise $c(t)$ with power spectral density

$$R_c(f) = R_w(f)\,|V(f)|^2 = \frac{N_0}{2}\,|V(f)|^2$$

- For an ideal bandpass filter $v(t)$ with bandwidth W the spectrum is shown below:





Filtered Gaussian Noise

- Since $R(f)$ is constant within the bandwidth W, such a process $c(t)$ is usually referred to as "white" bandpass process
- Let the noise process $c(t)$ be sampled at some time $t = t_0$. Then the sample value $c(t_0)$ is a Gaussian random variable with

$$p(c) = \frac{1}{\sqrt{2\pi\,\sigma^2}}\,e^{-(c-m)^2/2\sigma^2}$$

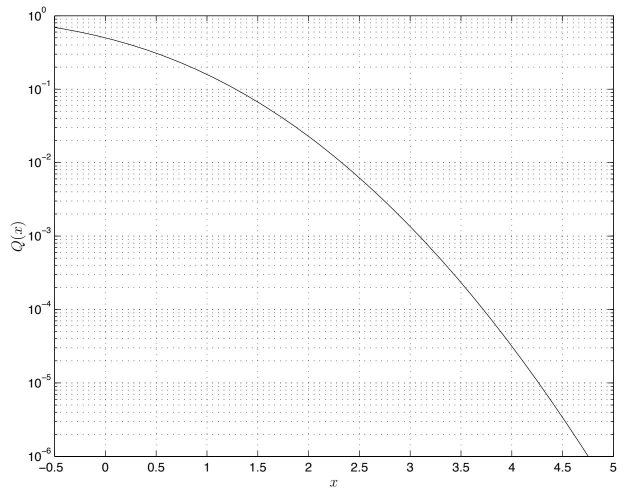
with mean $m = 0$ and variance $\sigma^2 = N_0/2\;E_v = N_0\,W = \mathcal{P}_c$
- Our bit error probability is related to the probability that the noise value $c(t_0)$ is larger than some threshold A

$$Pr\{c(t_0) \geq A\} = Pr\left\{\frac{c(t_0)-m}{\sigma} \geq \frac{A-m}{\sigma}\right\} = Q\left(\frac{A-m}{\sigma}\right)$$

- The $Q(x)$-function is defined as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}\,e^{-y^2/2}\,dy = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

The $Q(x)$-function



The $Q(x)$-function

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.0	5.0000e-01	3.0	1.3499e-03	6.0	9.8659e-10	9.0	1.1286e-19
0.1	4.6017e-01	3.1	9.6760e-04	6.1	5.3034e-10	9.1	4.5166e-20
0.2	4.2074e-01	3.2	6.8714e-04	6.2	2.8232e-10	9.2	1.7897e-20
0.3	3.8209e-01	3.3	4.8342e-04	6.3	1.4882e-10	9.3	7.0223e-21
0.4	3.4458e-01	3.4	3.3693e-04	6.4	7.7688e-11	9.4	2.7282e-21
0.5	3.0854e-01	3.5	2.3263e-04	6.5	4.0160e-11	9.5	1.0495e-21
0.6	2.7425e-01	3.6	1.5911e-04	6.6	2.0558e-11	9.6	3.9972e-22
0.7	2.4196e-01	3.7	1.0780e-04	6.7	1.0421e-11	9.7	1.5075e-22
0.8	2.1186e-01	3.8	7.2348e-05	6.8	5.2310e-12	9.8	5.6293e-23
0.9	1.8406e-01	3.9	4.8096e-05	6.9	2.6001e-12	9.9	2.0814e-23
1.0	1.5866e-01	4.0	3.1671e-05	7.0	1.2798e-12	10.0	7.6199e-24
1.1	1.3567e-01	4.1	2.0658e-05	7.1	6.2378e-13		
1.2	1.1507e-01	4.2	1.3346e-05	7.2	3.0106e-13		
1.3	9.6800e-02	4.3	8.5399e-06	7.3	1.4388e-13		
1.4	8.0757e-02	4.4	5.4125e-06	7.4	6.8092e-14		
1.5	6.6807e-02	4.5	3.3977e-06	7.5	3.1909e-14		
1.6	5.4799e-02	4.6	2.1125e-06	7.6	1.4807e-14		
1.7	4.4565e-02	4.7	1.3008e-06	7.7	6.8033e-15		
1.8	3.5930e-02	4.8	7.9333e-07	7.8	3.0954e-15		
1.9	2.8717e-02	4.9	4.7918e-07	7.9	1.3945e-15		
2.0	2.2750e-02	5.0	2.8665e-07	8.0	6.2210e-16		
2.1	1.7864e-02	5.1	1.6983e-07	8.1	2.7480e-16		
2.2	1.3903e-02	5.2	9.9644e-08	8.2	1.2019e-16		
2.3	1.0724e-02	5.3	5.7901e-08	8.3	5.2056e-17		
2.4	8.1975e-03	5.4	3.3320e-08	8.4	2.2324e-17		
2.5	6.2097e-03	5.5	1.8990e-08	8.5	9.4795e-18		
2.6	4.6612e-03	5.6	1.0718e-08	8.6	3.9858e-18		
2.7	3.4670e-03	5.7	5.9904e-09	8.7	1.6594e-18		
2.8	2.5551e-03	5.8	3.3157e-09	8.8	6.8408e-19		
2.9	1.8658e-03	5.9	1.8175e-09	8.9	2.7923e-19		

$Q(1.2816) \approx 10^{-1}$	$Q(5.1993) \approx 10^{-7}$
$Q(2.3263) \approx 10^{-2}$	$Q(5.6120) \approx 10^{-8}$
$Q(3.0902) \approx 10^{-3}$	$Q(5.9978) \approx 10^{-9}$
$Q(3.7190) \approx 10^{-4}$	$Q(6.3613) \approx 10^{-10}$
$Q(4.2649) \approx 10^{-5}$	$Q(6.7060) \approx 10^{-11}$
$Q(4.7534) \approx 10^{-6}$	$Q(7.0345) \approx 10^{-12}$

Chapter 4: Receivers

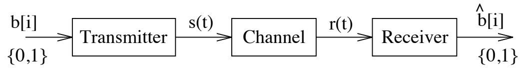
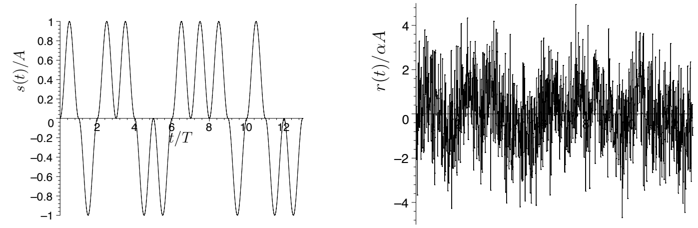


Figure 4.1: A digital communication system.



- How can we estimate the transmitted sequence?
- Is there an optimal way to do this?

Bit error probability

- Because of the noise the receiver will sometimes make errors
- During a time interval τ we transmit the sequence $\mathbf{b}$ of length

$$B = R_b\,\tau$$

- The detected (estimated) sequence $\hat{\mathbf{b}}$ will contain B_{err} bit errors

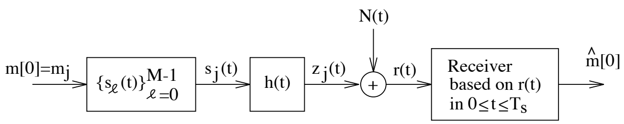
$$B_{err} = d_H(\mathbf{b}, \hat{\mathbf{b}}) \leq B$$

- The Hamming distance $d_H(\mathbf{b}, \hat{\mathbf{b}})$ is defined as the number of positions in which the sequences are different
- The bit error probability P_b is defined as

$$P_b = \frac{1}{B}\sum_{i=1}^{B} Pr\{\hat{b}[i] \neq b[i]\} = \frac{E\{d_H(\mathbf{b}, \hat{\mathbf{b}})\}}{B}$$

- It measures the average number of bit errors per detected (estimated) information bit

The Detection Problem



Assumptions:

- A random (i.i.d.) sequence of messages $m[i]$ is transmitted
- There are $M = 2^k$ possible messages, i.e., k bits per message
- All signal alternatives $z_\ell(t)$, $\ell = 1, \ldots, M$ are known by the receiver
- T_s is chosen such that the signal alternatives $z_\ell(t)$ do not overlap
- $N(t)$ is additive white Gaussian noise (AWGN) with $R_N(f) = N_0/2$

Questions:

- How should decisions be made at the receiver?
- What is the resulting bit error probability P_b?

An optimal decision strategy

- Suppose we want to minimize the symbol error probability P_s
- That means we maximize the probability of a correct decision

$$Pr\{m = \hat{m}(r(t)) \mid r(t)\}$$

where m denotes the transmitted message

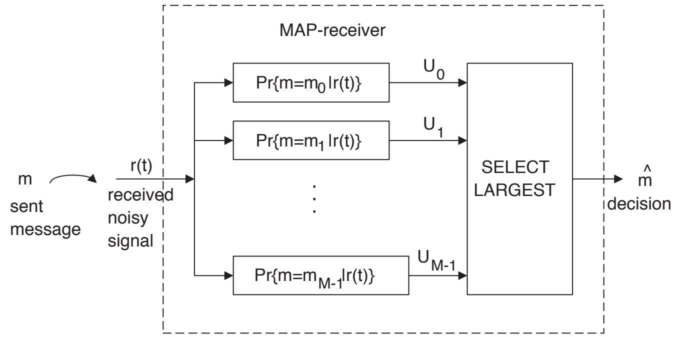
- This leads to the following decision rule:

$$\hat{m}(r(t)) = m_\ell \ ,$$
$$\text{where} \quad \ell = \arg\max_i Pr\{m = m_i | r(t)\}$$

- We decide for the message that maximizes the probability above
- A receiver that is based on this decision rule is called maximum-a-posteriori probability (MAP) receiver

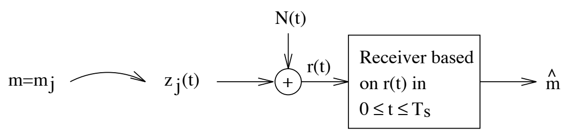
Structure of the general MAP receiver

- We know that one of the M messages must be the best
- Hence we can simply test each m_ℓ, $\ell = 0, 1, \dots, M-1$



This receiver minimizes the symbol error probability P_s

A slightly different decision strategy



- Instead of computing posterior probabilities, we can check which waveform $z_\ell(t)$ is most similar to the received signal $r(t)$
- A measure of similarity is the squared Euclidean distance

$$D_{r,i}^2 = \int_0^{T_s} \left(r(t) - z_i(t)\right)^2 \, dt = \int_0^{T_s} r^2(t) - 2\, r(t)\, z_i(t) + z_i^2(t) \, dt$$
$$= E_r - 2 \int_0^{T_s} r(t)\, z_i(t) \, dt + E_i$$

- A signal alternative $z_i(t)$ is similar to $r(t)$ if $D_{r,i}^2$ is small

The receiver needs to know the channel!

The Minimum Euclidean Distance Receiver

- The received signal is compared with all noise-free signals $z_i(t)$
- The message is selected according to the following decision rule:

$$\hat{m}(r(t)) = m_\ell \ ,$$
$$\text{where} \quad \ell = \arg\min_i \ D_{r,i}^2$$

- **Remark:**
  for equally likely messages, $P_i = 1/M$, $i = 0, 1, \dots, M-1$,
  this receiver is equivalent to the MAP receiver
- An implementation is often based on correlators with output

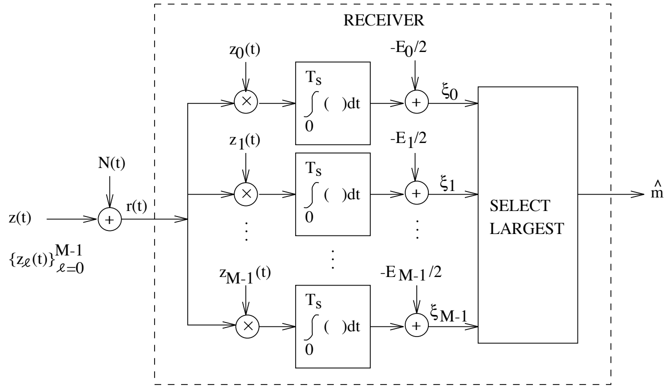
$$\int_0^{T_s} r(t)\, z_i(t) \, dt \ , \quad i = 0, 1, \dots, M-1$$

- We can write

$$\ell = \arg\min_i \ D_{r,i}^2 = \arg\max_i \int_0^{T_s} r(t)\, z_i(t) \, dt - E_i/2$$

The Minimum Euclidean Distance Receiver

Correlation based implementation:



For M-ary constellations with fixed pulse shape $g(t)$ the implementation can be further simplified

Example 4.4: 64-QAM receiver

*Assume that $\{z_\ell(t)\}_{\ell=0}^{M-1}$ is a 64-ary QAM signal constellation. Draw a block-diagram of a minimum Euclidean distance receiver that uses only **two** integrators.*

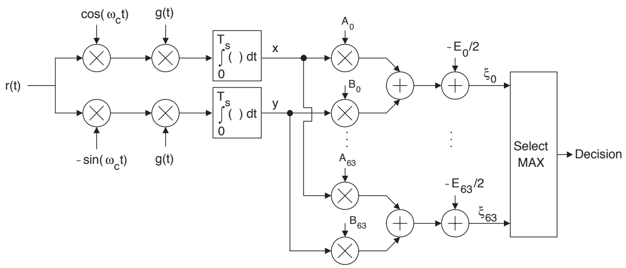
Solution:
A QAM signal alternative can be written as $z_i(t) = A_i g(t)\cos(\omega_c t) - B_i g(t)\sin(\omega_c t)$, where $g(t)$ is a baseband pulse. The output value from the i:th correlator in Figure 4.8 is,

$$\int_0^{T_s} r(t) z_i(t) dt = A_i \underbrace{\int_0^{T_s} r(t) g(t)\cos(\omega_c t) dt}_{x} - B_i \underbrace{\int_0^{T_s} r(t) g(t)\sin(\omega_c t) dt}_{-y} = $$
$$= A_i x + B_i y$$

Observe that x and y do not depend on the index i.
*Hence, a possible implementation of the receiver is to **first** generate x and y, and then calculate the M correlations $A_i x + B_i y$, $i = 0, i, \dots, M-1$. By subtracting the value $E_i/2$ from the i:th correlation, the decision variables $\xi_0, \dots, \xi_{M-1}$ are finally obtained.*

Example 4.4: 64-QAM receiver

The implementation of this receiver is shown below:



The complexity of this receiver is significantly reduced compared to the receiver in Figure 4.8 on page 241! Only two integrators are here used, instead of 64 (= M) in Figure 4.8.

- pulse shape and carrier waveform are recreated at the receiver
  $\Rightarrow$ these parts are very similar to the transmitter
- integration and comparison can be performed separately

A geometric interpretation

- Our receiver computes: (maximum correlation)

$$\max_i \{xA_i + yB_i - E_g/2\}$$

- Equivalently we can compute: (minimum Euclidean distance)

$$\min_i \left\{ \left(x - \frac{A_i E_g}{2}\right)^2 + \left(y - \frac{B_i E_g}{2}\right)^2 \right\}$$

Ex. QPSK: received point (x, y) is closest to the point of message m_3

x = message points, $\bullet$ = noisy received values (x, y)