Problems Week 7

7.1 We want to transmit the binary sequence

$$\mathbf{b} = 1\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 1$$

with bandpass 4-ary PAM signaling, using a rectangular pulse $g_{rec}(t)$ with amplitude 1 and duration $T = T_s$. Consider the following mapping from bits to the amplitudes $A_0 = -3$, $A_1 = -1$, $A_2 = +1$, and $A_3 = +3$:

$$00 \to A_0\ , \quad 10 \to A_1\ , \quad 11 \to A_2\ , \quad 01 \to A_3\ .$$

a. Draw the baseband transmit signal $x_I(t)$.

b. Assume a homodyne receiver with phase error $\phi_{err}(t) = -67.5°$(i.e.,$-3/8\pi$) and $A = 2$ for an ideal noise-free channel, i.e., $y(t) = x_I(t)\ \cos(2\pi\, f_c\, t)$. Draw the inphase and quadrature component signals $u_I(t)$ and $u_Q(t)$ in the interval $0 \le t \le 2T_s$.

c. Let us now see the benefit of coherent reception. In the interval $0 \le t \le 2T_s$, draw the three signals $u_I(t)\cos(\phi_{err}(t))$, $u_Q(t)\sin(\phi_{err}(t))$, and $\hat{u}_I(t) = u_I(t)\cos(\phi_{err}(t)) - u_Q(t)\sin(\phi_{err}(t))$.

7.2 Consider the same scenario as in Problem 7.1, but with 4-ary QAM signaling using amplitudes $A_0 = -1$, $A_1 = +1$, $B_0 = -1$, and $B_1 = +1$ with the mapping:

$$00 \to A_0, B_0\ , \quad 10 \to A_1, B_0\ , \quad 11 \to A_1, B_1\ , \quad 01 \to A_0, B_1\ .$$

a. Draw the baseband transmit signals $x_I(t)$ and $x_Q(t)$.

b. The homodyne receiver with phase error $\phi_{err}(t) = -67.5°$(i.e.,$-3/8\pi$) and $A = 2$ sees now the input signal $y(t) = x_I(t)\ \cos(2\pi\, f_c\, t) - x_Q(t)\ \sin(2\pi\, f_c\, t)$. Draw the inphase and quadrature component signals $u_I(t)$ and $u_Q(t)$ in the interval $0 \le t \le 2T_s$.

c. Is it possible to see the original data in $u_I(t)$ and $u_Q(t)$?

7.3 Assume again $y(t) = x_I(t)\ \cos(2\pi\, f_c\, t) - x_Q(t)\ \sin(2\pi\, f_c\, t)$ at the input of a homodyne receiver.

a. Use equations (3.171) and (3.172) to compute the signal

$$\hat{u}_I(t) = u_I(t)\cos(\phi_{err}(t)) - u_Q(t)\sin(\phi_{err}(t))\ .$$

What can we conclude?

b. *(Optional)* Can we recover the signal $u_Q(t)$ in a similar way?

7.4 For an ideal channel without noise the baseband signal produced by a homodyne receiver can be written as

$$\tilde{u}(t) = \frac{\tilde{x}(t)}{2} A \cdot e^{-j\phi_{err}(t)} = u_I(t) + j\, u_Q(t)\ ,$$

where

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t)\ .$$

Show that this expression is equivalent to equations (3.171) and (3.172).

7.5 Let us now describe the signals in Problem 7.2 using complex baseband notation

$$\tilde{x}(t) = x_I(t) + j\ x_Q(t) = \sum_{n=0}^{4} \tilde{A}_{m[n]}\, g(t - nT_s)$$

a. Determine the sequence $\tilde{A}_{m[n]}$.

b. Draw the values $\tilde{A}_{m[n]}$ as points in the complex plane and label each point with the corresponding input bits.

c. The sequence is transmitted over a channel with $\tilde{h}(t) = 2\,\tilde{\alpha}\,\delta(t)$ and additive noise $\tilde{w}(t)$, where $\tilde{a} = 0.001\, e^{-j\phi_\alpha}$. At the receiver the signal $\tilde{u}(t)$ is match-filtered, sampled and multiplied with a scaling coefficient $\tilde{C}$, resulting in the decision variable

$$\tilde{\xi}_{m[n]} = \tilde{C}\left(\tilde{A}_{m[n]}\,\tilde{\alpha}\,e^{-j\phi_{err,n}}\frac{A\,E_g}{2} + \tilde{\mathcal{N}}_n\right)\ ,$$

where $\tilde{\mathcal{N}}_n$ denotes the contribution of noise at the output of the matched filter. The coefficient $\tilde{C}$ is chosen such that $\tilde{\xi}_{m[n]} = \tilde{A}_{m[n]}$ for $\tilde{\mathcal{N}}_n = 0$. Determine $\tilde{C}$.

d. Assume now that another sequence $\mathbf{b}$ is transmitted under the same conditions. The decision variables computed by the receiver are

$$\tilde{\xi}_{m[n]} = (0.5 + 0.5\,j),\ (-1.5 + 0.5\,j),\ (1.5 + 1.5\,j),\ (-0.5 + 0.5\,j),\ (-1.5 - 0.5\,j)$$

Draw the values $\tilde{\xi}_{m[n]}$ as points in the complex plane, together with the possible ideal values $\tilde{A}_{m[n]}$. For a minimum Euclidean distance receiver, determine the estimated sequences $\tilde{A}_{m[n]}$ and $\hat{\mathbf{b}}$.