# Solutions to exercise 1 in <br> EITF25 Internet - Techniques and Applications - 2014 <br> Physical layer (OSI 1) 

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## 1

a
If a bit occupies 1 millisecond $=10^{-3}$ seconds, then a second is occupied by $\frac{1}{10^{-3}}=10^{3}$ bits $=1 \mathrm{kbps}$.

## Answer: 1 kbps

b
If a bit occupies 2 microseconds $=2 \times 10^{-6}$ seconds, then a second is occupied by $\frac{1}{2 \times 10^{-6}}=0.5 * 10^{6}$ bits $=500 \mathrm{kbps}$.

Answer: 500 kbps
2
a
If bits are produced/received at a rate of $100 * 10^{3}=10^{5}$ per second, then one bit occupies $\frac{1}{10^{5}}=10^{-5}$ seconds $=10$ microseconds.

Answer: 10 microseconds
b
If bits are produced/received at a rate of $2 \times 10^{6}$ per second, then one bit occupies $\frac{1}{2 \times 10^{6}}=0.5 \times 10^{-6}$ seconds $=0.5$ microseconds.

Answer: 0.5 microseconds

## 3

## a

In a NRZ (Non-Return-to-Zero) modulated transmission ones and zeros are a represented by a specific output level, constrained to a specified duration, see figure 1 .


Figure 1: NRZ bit vs. voltage level assignment
As a consequence, unless you know when the transmission started and the duration a bit occupies, you will be unable to distinguish between multiple consecutive bits of the same sign. In a communication system, both receiver and transmitter thus need to be synchronized.

b
In a Manchester modulated signal, each bit i demarked by an output transition during the allocated bit duration (symbol) as opposed to a constant level seen in NRZ modulation. In Figure 2 a zero is represented by a falling edge, while a one is represented by a rising edge.


Figure 2: Machester bit representation as according to IEEE 802.3
As each bit infers a output level transition, with Manchester coding, each bit can be identified without prior clock synchronization. Nevertheless, both receiver and trans-
mitter needs to agree on which transition to represent which bit. Moreover, In this case, we are dealing with a sequence of all zeros, the signal is thus modulated with all falling edges for each bit. Manchester coding has a relative low throughput compared to more complex modulation schemes and is today mainly used in systems such as 10BASE-T ethernet (IEEE 802.3) and NFC.

c
As you will see in problem 5, Differential Manchester coding deals with transitions between symbols rather than output levels within a symbol. A sequence with all zeros is represented by the absence of symbol transitions, and the waveform is therefore identical to the Machester coded one in problem 3b.


## 4

When applying the Manchester modulation scheme presented in Figure 2 to the observed sequence, we arrive at the bit sequence presented in Figures 3, 4, and 5.

## a

Following the above convention yields.


Figure 3: Decoded 4a Manchester waveform

## Answer: 0100110001

b
Following the above convention yields.


Figure 4: Decoded 4b Manchester waveform

## Answer: 1100110110

c
Following the above convention yields.


Figure 5: Decoded 4c Manchester waveform

## Answer: 1000011100

## 5

As opposed to Manchester coding, Differential Manchester coding only represents transitions between $0 \rightarrow 1$. See Figure 6 .


Figure 6: Transitions
As such, several transitions represent just as many ones. A single transition followed by a consecutively repeated symbol represents a one followed by zeros, until the next transition. As such, the absence of a transition signifies a zero. See Figure 7


Figure 7: Sequence with transitions

When observing an arbitrary waveform, as we do not know whether it is a continuation of the previous symbol or a transition, the first bit is inherently unknown. See Figure 8.


Figure 8: Decoding the first bit
As you can see in Figure 8, we do not know if first symbol is zero that transitions into a one, or the tail of a sequence of transitions representing several ones. Differential Manchester is used because a transition is less likely to be misinterpreted than a individual symbols by the receiver, given a noisy channel. Differential Manchester encoding is predominantly used in magnetic and optical storage.
a
Following the above convention yields.


Figure 9: Decoded 5a Differential Manchester waveform.

## Answer: 110101001

b
Following the above convention yields.


Figure 10: Decoded 5b Differential Manchester waveform.

## Answer: 010101101

c
Following the above convention yields.


Figure 11: Decoded 5c Differential Manchester waveform.

```
Answer: 100010010
```


## 6

## a

Following the convention established in problem 4.

b
Following the convention established in problem 5. The first symbol is to illustrate the state of the signal.


## 7

Being it a wireless, an optical, or an electrical link, in FDM (Frequency Division Multiple access) the entire or a portion of that mediums frequency spectrum bandwidth is divided into channels, see Figure 12. Moreover, this is one approach for accommodating multiple connections in a common link, where the transmitter ( Tx ) and receiver ( Rx ) are transmitting and receiving in the same allocated frequency range, channel. In most FDM based communication systems, a frequency guard interval is introduced to separate the channels, see Figure 12b. This is to ensure that any leakage from one channel is not interfering with its adjacent channels. The guard interval does not carry any intentional information. Note that guard intervals are only needed to separate the channels in the medium, and not the channels from beyond the frequency range of the medium. Unless there is another communication link at a frequency range above or below, in which case, a guard interval is introduced between the links. However, this is not taken into account when referring the bandwidth of the observed link.


Figure 12: Link multiplexed across 4 frequency channels

In this instance, it is specified that each channel needs a bandwidth of $4000 \mathrm{~Hz}=$ 4 KHz with a guard interval of $200 \mathrm{~Hz}=0.2 \mathrm{kHz}$. SInce there are five channels, we need four guard intervals to separate them. Moreover, this leaves us with a total link bandwidth of $5 \times 4 \mathrm{kHz}+4 \times 0.2 \mathrm{kHz}=20.8 \mathrm{kHz}$. See Figure 13


Figure 13: Link bandwidth breakdown, guard intervals are shaded (Not to proportion)
Answer: 20.8 kHz

## 8

The bit rate from DVD (in average):

$$
R_{b, D V D}=\frac{8 e 9 * 8}{2 * 60 * 60}=8.9 \mathrm{Mb} / \mathrm{s}
$$

Bit rate from uncompressed source:

$$
R_{b, \text { uncomp }}=25 * 720 * 576 * 3 * 8=249 \mathrm{Mb} / \mathrm{s}
$$

Compression $=249 / 8.9=28$. Without compression the time on a DVD would be $2 *$ $60 / 28=4.28$ minutes. (Alternative derivation $8 e 3 * 8 /(249 * 60)=4.28$ minutes).

## 9

## a

The breaking frequency can e.g. be set to $f_{0}$, and the amplitude becomes half the amplitude of the original signal.
b
Set $F_{s}=f_{0}$ and filter with a $L_{P}$ filter breaking at $F_{s}$.

## 10

a
The input to the quantiser is the continuous variable $X$ which is mapped to the quantiser output $X_{Q}$. In a linear quantiser the size of the steps is constant, say $d$.


Figure 14: A linear quantisation function with $M$ levels.


Figure 15: The quantisation error, $X-X_{Q}$, and distortion, $d(x, x Q)=\left(X-X_{Q}\right)^{2}$, for the linear quantisation function in Figure 14

Estimate the average distortion by conditioning on level k the distortion is:

$$
E\left[\left(X-X_{Q}\right)^{2} \mid k\right]
$$

Viewing the distortion as a noise is convenient to consider since the signal has zero mean.

$$
E\left[X^{2} \mid X_{Q}=0\right]=\int_{-d / 2}^{d / 2} \frac{x^{2}}{d} d x=\frac{d 2}{12}
$$

Averaging over all levels

$$
E\left[\left(X-X_{Q}\right)^{2}\right]=\sum_{k=0}^{M-1} E\left[\left(X-X_{Q}\right)^{2} \mid k\right] \frac{1}{M}=\sum_{k=0}^{M-1} \frac{1}{M} \frac{d^{2}}{12}=\frac{d^{2}}{12}
$$

b
$E\left[X^{2}\right]=\int_{-M d / 2}^{M d / 2} x^{2} \frac{1}{M d} d x=(M d) \frac{2}{12}=>S Q N R=M=2^{k}$ and $S Q N R_{d B}=2^{k} 10 \log 2 \approx k 6 d B$

