Synchronization for OFDM systems

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Synchronization: offset/errors and their cause

- **Symbol (timing) offset (OFDM and DMT)**
  - transmitter and receiver do not have a common time reference
  - receiver needs to find symbol boundaries to avoid ISI/ICI

- **Carrier frequency/phase offset (OFDM)**
  - carrier frequency/phase of transmitter’s local oscillator (LO) and receiver’s LO can be off by some ppm
  - resulting frequency difference $\Delta F_c \text{ Hz}$ between transmitter’s and receiver’s carrier introduces the additional term $e^{j2\pi \Delta F_c / F_s n}$ in the baseband multiplex $\rightarrow$ ICI
  - receiver needs to compensate for the frequency offset (and the phase offset for coherent detection)

- **Symbol clock (sampling frequency) offset (DMT and OFDM)**
  - frequency/phase of transmitter’s local oscillator (LO) and receiver’s LO for the sampling clock can be off by some ppm
  - resulting frequency difference $\Delta F_s \text{ Hz}$ between transmitter’s and receiver’s clock causes
    - a gradually growing timing offset $\rightarrow$ ISI/ICI
    - a slightly too large (or too small) subcarrier-spacing $\rightarrow$ ICI
  - receiver needs to compensate for the clock offset
Symbol (timing) offset

- receiver assumes symbols on the dots in the time-frequency grid
- effect: severe ISI and ICI
Carrier frequency/phase offset

receiver assumes symbols on the dots in the time-frequency grid

effect: ICI
Carrier frequency/phase offset cont’d

Assume a $\Delta F_c$ between the carrier frequency of transmitter and receiver:

Fourier transforms of the carriers

- $f_1$
- $f_2$
- $f_3$
- $f_4$
- $f_5$
Consequence of $\Delta F_c \neq 0$:

- each subcarrier of a multicarrier symbol ends up on a frequency position that is shifted by $\Delta F_c$ Hz compared to the transmitter → orthogonality of the subcarriers is gone

- impact of a normalised frequency offset $\Delta_c = \Delta F_c / (F_s / N)$ (normalised by the subcarrier spacing) on the $\ell$th receive symbol is described by

$$y_{\ell} \approx x_{\ell} \cdot \underbrace{\text{sinc} (\Delta_c) e^{j \pi \Delta_c}}_{\text{scaling, I/Q cross-coupling}} + \sum_{k \neq \ell} x_k \cdot \text{sinc} (k - \ell + \Delta_c) e^{j \pi (k-\ell+\Delta_c)}$$

ICI caused by carrier offset

From (1) it is clear why carrier synchronization in OFDM is critical: interference from all the other subcarriers occurs if $\Delta F_c \neq 0$. 

\[(1)\]
regarding the carrier offset correction, we distinguish

- controlled oscillator; the oscillator of the down-converter (mixer) is adaptively tuned such that it corrects the carrier offset right away;
- freely running oscillator followed by correction; the oscillator is not tunable, which simplifies its implementation; the resulting carrier offset is corrected by a multiplication of the receive signal multiplex with $e^{-j2\pi\Delta F_c/F_s n}$ before the DFT block;

parameter estimation in many practical implementations is based on correlation

most schemes operate in stages

- coarse lock ("acquisition")
- fine tuning of the offset ("tracking")

most robust methods, especially in fading environments, employ pilot symbols
Clock (sampling frequency) offset

\[(N + L)\Delta T_s\]

- receiver assumes symbols on the dots in the time-frequency grid
- effects: gradually increasing ICI (with carrier No.) due to mismatch in carrier spacing, gradually increasing symbol-timing offset
Assume a difference $\Delta F_s = F_s^{(\text{transmitter})} - F_s^{(\text{receiver})}$ between the sampling clock of the transmitter and the receiver:

Fourier transforms of the carriers
Clock (sampling frequency) offset cont’d

Consequence of $\Delta F_s \neq 0$:

- frequency spacing of receive signal’s DFT is larger or smaller than the spacing of the transmit signal’s DFT $\rightarrow$ orthogonality is lost

- impact of a normalised sampling clock offset $\Delta_s = \Delta F_s / F_s^{(receiver)}$ on the $\ell$th receive symbol is described by

\[
\begin{align*}
y_{\ell} & \approx x_{\ell} \frac{e^{j2\pi\ell\Delta_s} - 1}{j2\pi\ell\Delta_s} + \sum_{k \neq \ell} x_k \frac{e^{j2\pi(k(1+\Delta_s) - \ell)} - 1}{j2\pi(k(1 + \Delta_s) - \ell)} \\
\Delta_s & \ll \approx x_{\ell} \underbrace{(1 - j\pi\ell\Delta_s)}_{\text{scaling, I/Q cross-coupling}} + \sum_{k \neq \ell} x_k \frac{k\Delta_s}{k - \ell} \tag{2}
\end{align*}
\]

- many terms contribute to the inter-carrier interference
- tolerable distortion for a subchannel depends on constellation size and desired bit error rate performance
- in practice, system clock is often locked to carrier-frequency clock
Synchronization

synchronization consists of two tasks:

- estimation of an appropriate parameter (frequency offset, phase offset, time offset)
- actual offset correction based on the estimate

regarding the estimation task, we distinguish

- methods that are supported by deliberately inserted synchronization-assisting signals (pilot signals, synchronization symbols)
- methods that operate without this assistance (often referred to as “blind” estimation algorithms)
Symbol timing offset—CP-length=CIR-length

Channel impulse response

Channel input

Channel output

ISI/ICI-free symbol timing instant

unique ISI-free timing instant

Prefix and postfix of length $T^{(W)}$ are windowed cyclic extensions to improve out-of-band spectrum
Symbol timing offset—CP-length $> \text{CIR-length}$

- Channel impulse response
- Channel input
  - $T^{(W)}$, $T^{(CP)}$, $T^{(MC)}$, $T^{(W)}$
- Channel output
  - Falling transient of previous symbol
  - Rising transient of current symbol
  - Steady-state response of current symbol
  - ISI/ICI-free timing window of length $T^{(CP)} - \tau^{(CIR)}$

- Latest timing-instant (right-most in picture) yields largest delay-spread immunity
Symbol timing offset—CP-length $>\text{CIR-length}$ cont’d

- captured $n$ samples are cyclically shifted by $n$ samples
- cyclic time-domain of $r(n), n = 0, \ldots, N - 1$ by $s$ samples corresponds to phase rotation
  \[
r'(n) = r((n - s) \mod N) \quad \leftrightarrow \quad R'(k) = R(k) e^{-j2\pi sk/N}, \quad k = 0, \ldots, N - 1,
\]
  where $R(k)$ and $R'(k)$ denote the DFT of $r(n)$ and $r'(n)$
- differential modulation in time: inherently immune to time-invariant phase rotations
- absolute modulation: phase rotations are taken care of by frequency-domain equalization
Pilot-based timing/frequency synchronization: time-offset

- time-domain pilot symbol with 2 identical halves (period $P = N/2$)
- key idea: if CP is long enough, both halves are identical (up to noise) at channel output
- normalized correlation measure
  \[
  M_{\text{time}}(d) = \frac{\sum_{m=d}^{d+N/2-1} r^*(m)r(m+N/2)}{\sum_{m=d}^{d+N-1} |r(m)|^2}
  \]
  timing instant
  \[
  \hat{d} = \arg \max_d M_{\text{time}}(d)
  \]
  pinpoints the first sample of the pilot symbol’s steady-state part
receive signal: \( r(n) = r'(n) e^{j2\pi(n_0+n)\Delta F_c/F_s} \)

any two samples \( r(n) \) and \( r(n + P) \) are identical up to a constant phase difference

\[
\arg(r(n + P)) - \arg(r(n)) = \arg(r(n + P) r^*(n)) = 2\pi P \Delta F_c / F_s + 2\pi i
\]

and noise

fractional part of frequency offset:

\[
\hat{\Delta F}_c = \frac{F_s}{2\pi P} \arg(\sum_n r(\hat{d} + n + P) r^*(\hat{d} + n))
\]

if \( |\Delta F_c| \leq \frac{F_s}{2P} \), \( \Delta F_c \) yields correct offset

fractional frequency-offset is corrected before FFT processing through counter-rotation yielding \( r'(n) = r(n) e^{-j2\pi n \Delta F_c / F_s} \)
Pilot-based timing/frequency synchronization: frequency-offset cont’d

- for integer part of frequency offset: second training symbol $R_2$ with differentially modulated data with respect to the pilot symbol $R_1$
- pilot symbols are ICI-free but shifted by $iF_s/P$
- differential modulation: $R_2 = R_1 P$, where $P$ is pseudo-random number (PN) vector
- similarity between $R_2(k)/R_1(k)$ and a shifted version $P(k - iN/P)$ of $P$ is measured by

$$M_{\text{freq}}(i) = \sum_k \left| \frac{R_2(k)}{R_1(k)} P(k - iN/P) \right| = \sum_k \left| \frac{R_2(k)R_1^*(k)P(k - iN/P)}{|R_1(k)|^2} \right|$$

and its maximum indicates the integer part

$$\hat{i} = \arg \max_i M_{\text{freq}}(i)$$

of the frequency offset

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1Note that the sum index $k$ includes only subcarriers for which $R_1(k) \neq 0$ and $k - iN/P \in [1, N]$. 
CP-based timing/frequency synchronization: time-offset

- A longer-than-necessary CP provides repetitive pattern of receive stream
- Correlation measure
  \[ \gamma(d) = \sum_{m=d}^{d+L-1} r^*(m) r(m + N) \]
- Power measure
  \[ \Phi(d) = \frac{1}{2} \sum_{m=d}^{d+L-1} |r(m)|^2 + |r(m + N)|^2 \]
- Joint ML estimation of both timing offset and frequency offset yields the estimate
  \[ \hat{d} = \arg \max \{|\gamma(d)| - \rho \Phi(d)| + L, \]
  which pinpoints the first sample of the pilot symbol’s steady-state part
- \( \rho \) is the magnitude of correlation coefficient between two \( N \)-spaced samples
- Performance degrades with stronger time dispersion and with increasing channel noise
CP-based timing/frequency synchronization: frequency-offset

- same principle as before: repetitive signal parts are identical up to a phase difference
- joint ML estimation of offsets in time and frequency yields

$$\hat{\Delta F_c} = \frac{F_s}{2\pi N} \arg(\gamma(\hat{d}))$$

- performance degrades with stronger time dispersion and with increasing channel noise
Tracking

- After acquisition, time offsets and frequency offsets are updated on a regular basis.
- Dedicated tracking pilots are used.
- Timing offset: linearly increasing phase offset across subcarriers.
- Frequency offset: linearly increasing phase offset over time.
Synchronization in OFDMA-UL

- OFDMA idea: separate users via subcarrier-assignment (FDMA)
- subcarrier allocation: subband allocation, interleaved allocation, general (unstructured) allocation
- signals from different users arrive with different timing-offsets and frequency-offsets
- chicken-egg problem
  - synchronization is required for orthogonality
  - orthogonality is required to correct offsets
OFDMA-UL timing schemes

**Quasi-synchronous timing scheme**

blocks transmitted by users:

\[ T^{(CP)} \quad T^{(MC)} \]

blocks received by BS:

\[ t_0 \quad t_1 \quad t_0 + T^{(CP)} \]

**Synchronous timing scheme**

blocks transmitted by users:

\[ t_0 \quad t_0 + (\tau_2^{(DL)} + \tau_2^{(UL)}) \]

blocks received by BS:

\[ t_0 \quad t_0 + T^{(CP)} \]

- Quasi-synchronous scheme: \( T^{(CP)} \geq \max_k \tau_k^{(UL)} + \max_k \tau_k^{(DL)} + \max_k \tau_k^{(CIR)} \)
- Synchronous scheme: \( T^{(CP)} \geq \max_i \tau_i^{(CIR)} \)
OFDMA-UL with subband allocation

- time offset: synchronous or quasi-synchronous timing scheme
- frequency-offset estimation and correction: separation in frequency
- direct frequency-offset correction:

\[ e^{-j2\pi n \hat{\epsilon}_1 / N} \]
\[ e^{-j2\pi n \hat{\epsilon}_K / N} \]

\[ r \]

\[ R_1 \]
\[ R_K \]
OFDMA-UL with subband allocation

- time offset: synchronous or quasi-synchronous timing scheme
- frequency-offset estimation and correction: separation in frequency
- offset correction in FFT-domain (lower complexity):

\[
\begin{align*}
R_1 & \text{ circular convolution with } C_1 \\
R_K & \text{ circular convolution with } C_K \\
R & \text{ FFT}
\end{align*}
\]
Summary

- Different types of physical-layer offsets:
  - symbol (timing) offset
  - carrier frequency/phase offset
  - clock (sampling frequency) offset
- Single-user pilot-based synchronization
- Single-user CP-based synchronization
- Synchronization in OFDMA UL: challenge; approach for subband allocation